Population Review

Volume 44, Number 2, 2005

Type: Article pp. 24-49

The Maltho-Marxian Hypothesis 'Economics Controls Population': A Test and a Projection

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Abstract

Malthus and Marx held that population was controlled by economics. Malthus believed there were environmental constraints on the supportable population; Marx felt that human ingenuity would overcome all limits to growth. Neither had supporting data. The US Census reveals an intermediate position in which the supportable limit increases exponentially (faster than Malthus expected) but at 1/4 the rate of unfettered human reproduction (slower than Marx expected). Its rate offers an independent estimate of effective economic growth. A brief and sharp-cornered excursion from the resulting theoretical line forms a nearly perfect Gaussian dip, with the Depression on the down side and the Baby Boom on the other. A related analysis shows that the world population is well fitted by a 'Pimentel logistic' stabilizing at 2-3 billion after an 'oil-supported' Gaussian bulge. This approach explains the Doomsday hyperbola, and also what allows us to avoid its singularity.

Keywords

US census, economic factors, logistic model, Malthusianism, Marxism, population dynamics, population projection, theoretical models

*This paper is dedicated to the memory of Dr. Garrett Hardin, friend and mentor, who lived and died by his convictions.

Introduction

The relation between economics and population is both reciprocal and vexed, and good data are scarce. The US Census presents perhaps the only available example in which economic control is clearly demonstrable. Despite large fertility differences among cohorts, the overall population growth provides what appears to be the best available estimate of effective economic growth (somewhat lower than official estimates). Because the US has been the nation least affected by Malthusian miseries, it provides valuable evidence of the effectiveness (and need for) Malthusian vice as the means by which the population adjusts itself painlessly to economic conditions.

If the world's dominant nation shows economic control of population, what of the world as a whole? Malthusian misery has been a major world control. Warfare, democide, plague, famine, malnutrition, poverty, and ignorance are painful ways to adjust the population, but they keep the growth rate well below the biological potential. The rate achieved shows a strict dependence upon economics as represented by the energy available to modify our environment. This dependence subsumes all of the minor reasons that world growth has been faster than exponential (medical care, transport, international aid, &c). It also reveals just how vulnerable we are to decreases in energy availability.

Hypotheses and Models

Current demographic interest seems to focus on the social mediation of fertility and the use of short-term component-cohort equations (Lotka 1925) which recognize the variability within a society. The larger question that intrigues me is, What governs the results of social mediation of fertility? One man's noise is another man's data, and local fertility variation is noise in my data. Given the bias against the holistic approach, it seems useful to begin with two reminders. Firstly, Ahlburg (1995) showed that both simple overarching and complex detailed models work and answer different questions. Secondly, the objective of 'curve fitting' is not numbers but understanding: 'We have data; we seek insight and understanding. How do we go from one to the other? What's the connection?

'The answer to this question was Kepler's most important discovery. **Data are connected to understanding by a model.** When … the model … is mathematical ... not only does [it] lead directly to understanding but one may query the model to gain further information and insight.' (McLaughlin 1999: 19) (emphasis added).

Many people feel that models are unnecessary, but this is not the case. In the absence of an explicit rational model, people fall back on unexamined constructs jury-rigged from background information, anecdotes, propaganda, beliefs, and prejudice, which replace, however inadequately, the model as a basis for insight and decision making. This is particularly true of the politically sensitive field of demography.

The Maltho-Marxian model ('growth tracks economics') is more subtle than the dominant popular model known as 'Deevey's Staircase' ('punctuated equilibrium'). The Staircase seems largely unknown to demographers, and certainly none ever tried to correct it. Yet it appeared twice in

Scientific American (Deevey 1960, Kates 1994, and several times in trade and textbooks by authors who should have seen its errors (Ehrlich & Ehrlich 1970: Fig. 2.2, Livi-Bacci 1989; Fig. 1.11, J. Cohen 1995: Fig. 11.1). It claimed that the world population came into stable equilibrium three times, once with a hunting-gathering economy (wrong: rising population forced the development of agriculture (M. Cohen 1977)), again with agriculture (wrong: this 'equilibrium' was apparently an extrapolation of the European Black Death to the world), and finally with industry (a false impression resulting from a log (*past time*) axis, which never reaches the present moment and flattens all curves). At scientific meetings it has suppressed discussion of the disadvantages of overpopulation (e.g.: Scripps Institution of Oceanography's 'Human Population Density and the Quality of Life', La Jolla, 19-20 Feb 1970, was quashed by a slide showing the Staircase and the assertion of a fisheries biologist that there were no problems, so let's all go home). It was 41 years before the errors were published (Schulze & Mealy 2001). Others have described similar difficulties in getting unpopular research published in this field (Lambert &a 1988). This is surely a psychological bias attributable to DNA itself. In any other field the Staircase model would have been rejected immediately. Because it showed us what our DNA wants us to believe—that there is no such thing too much human DNA—none of the thousands of scientists who saw the graph caught the errors. It appears to have been taken as gospel by the staff of every politician in existence, and to have been the leitmotif of the 1994 United Nations International Conference on Population and Development in Cairo. A more accurate model might lead to clearer thinking.

When Malthus wrote (6th and final edition in 1826), he had no usable data with which to test his hypothesis. Although he was 9th wrangler at Cambridge in 1788, placing him among the world's best mathematicians, he resisted whatever temptation he might have felt to present his ideas in mathematical form. He described his hypothesis in words, which I paraphrase as: Food supply rises arithmetically [linearly with time]; population rises geometrically [exponentially with time]. When the two trends meet, the population is kept below the supportable maximum by moral restraint, misery or vice.

By 1826 Malthus had examined the population records of 24 nations (Malthus II.i-x). The only one then smooth enough to be fitted to any theoretical curve was that of the United States. As shown in Fig. 1B, it was a nearly perfect exponential. In the absence of the miseries ('unwholesome occupations, too severe labor, exposure to the elements, extreme poverty, bad nursing of children, great towns, excesses of all kinds, ... diseases, epidemics, war, plague, and famine' M.I.ii), and no need for moral restraint (chastity and delayed marriage M.IV.i,ii) or vice (delicately undefined and hence easily overlooked, but it was Regency code for family planning), Malthus's 'geometrical growth' was supported by the US data. He noted this but did not model it. Writing before the Industrial Revolution, he could see no way by which the food supply could be doubled more than a few times [M.I.i]. What he could not anticipate were advances in plant breeding and the brief availability of fossil energy for irrigation, cultivation and transport of food, and conversion of atmospheric nitrogen to fertilizer.

Two examples of the intersection of population and limit are known, both from islands. These were caused by the collapse of the limit onto an unsuspecting population, rather than by population growth itself. Ca. AD 1550, a millennium of easy living for the Rapanui ('Easter' Islanders) came to an abrupt end, apparently because they deforested their island (MacIntyre 1999). 'Misery' here meant obsidian spear-points and cannibalism (Bahn & Flenley 1992), and the population dropped to 1/3 of its peak. In 1845 the potato blight *Phytophthora infestans* caught up with its host in Ireland. Here, 'misery' meant starvation and emigration, and the population dropped to 1/2 of its peak. Neither collapse falsifies the idea that most cultures avoid exceeding the supportable limit, if necessary by infanticide and warfare, practiced by the parent population of Rapa Nui (Ferdon 1981: 143). Infant cannibalism had been famously recommended to the Irish by Swift (1729) a century before the collapse. The loss of 2/3 of the population is typical of population crashes in animal communities (Taylor 1970: 205).

Marx was more sanguine than Malthus. I find him impossible to read and so do not have a direct quotation. However, the party line ever after maintained that by replacing the exploitation of man by the exploitation of nature, population could increase indefinitely. For every new mouth, there was a new pair of hands to feed it. In short, man's ingenuity would see to it that the food supply (and the maximum supportable population) would grow geometrically also. To the best of my knowledge, Marx eschewed mathematics completely, and never thought of trying to model his ideas to compare them against the real world. Nevertheless, he saw that his theory of human nature was incompatible with Malthus's and attacked him rhetorically—but never with data. Bettany observed in 1890, 'of the multitudes who have denounced *Malthus on Population*, very few have read the book' (M.Intro.p. v). Responding to 20 years of criticism, Malthus complained that he could find little to reply to except 'absurdity, inconsistency, and unfounded assertion' (M. App. 1817). This has not changed. Nearly every comment on Malthus that I meet in lectures, the media. or the Internet calls him wrong because population is still growing. This shows that his message has not been understood. If you retain only one factoid from this paper, let it be this: If Malthus were wrong, the population of the US in 2006 would be 5 billion, 18 times larger than it actually is.

Analyzing the reason for wildly divergent future-population projections, Umpleby (1990) described 'different philosophies concerning the manipulation of data'. Demographers, he suggested, tend to examine the population, and birth, death, and migration rates (that is, the function and its first derivatives), paying particular attention to recent data with component-cohort models. The resulting equations can, with sophisticated mathematics, sufficient evanescent data, and luck, make projections good for a decade (Monro 1993). UN world projections on this basis are revised downward as input data changes (e.g.: 10-12.5 billion by 2050 in 1992, vs 7.8-10.6 billion in 2005). Physical scientists prefer to work with longer time series, attend to higher derivatives, and see instability where demographers assume stability. Thus Deevey (1960) took prehistoric estimates into account (even if his graph was flawed). Von Foerster (1961) felt that 2000 years of population history had some relevance to the next 50. I show that the 360 years of US Census data form a coherent data set. Fremlin (1964) was not embarrassed to base a 1000-year projection on the assumptions of continuation of historic behavior and the availability of fusion power by 2000 AD.

The Maltho-Marxian Hypothesis

The hypothesis to be tested is this: Populations are controlled by a logistic with an exponential limit which represents the 'maximum population supportable at the socially desired mean standard of living'. This is neither strict Malthusianism nor strict Marxism, but a hybrid of the type first formulated by W. E. Howland at Purdue. As you might expect, Howland never persuaded a journal to publish his work. The only record he left aside from pencilled nomograms, handwritten notes, and a typescript (personal communication) was a letter to *Science* (1961). He chose to examine the linear Malthusian limit, which unfortunately integrates to the transcendental exponential integral $Ei(x)$ (Abramowitz & Stegun 1965: §5.1). Before computers, this made his version of the problem much more difficult than necessary (hence the nomograms).

The Model

The differential form of the Maltho-Marxian hypothesis begins with the standard 'sigmoid' logistic equation (Verhulst 1845): $p' = rp(1 - p/K)$ where p' is the time derivative of the population, r is the growth rate per unit time, and K the limiting population. If we set $K = \infty$ we have the ordinary exponential where growth p' is proportional to the population at rate r. This works well for the initial phase of the growth of single-celled organisms in a pristine environment, and is apparently the usual assumption about economic growth: there is no limit. Hardin (1999: 135 ff) shows how deeply this Marxist hypothesis pervades current economic ideology. If we set K = constant, we recover Verhulst's assumption about animals, used by biologists for non-human populations. Pearl & Reed (1920) reinvented it to describe the US population. Here $\int p'dt$ forms a Staircase step (line F in Fig. 1A). Because this fits many animal populations, including *Drosophila* in a milk bottle on a fixed diet (Pearl 1924), we call it the 'fruitfly' logistic. Its r and K parameters are immortalized in population biology (MacArthur 1962), where the r-strategy of weeds and mice elects quantity breeding and the K-strategy of cetaceans and pachyderms elects few offspring and high-quality parental investment. The US confusion over abortion is the result of an unconscious *de facto* shift from an r-strategy prior to 1829 to a K-strategy after 1983 (Fig. 1B). Neither this change, or the need for it, is understood by the religious right.

If we set $K = a+bt$, we recover Malthus's linear approximation, investigated by Howland. (A number of other equations have carried this name, e.g., Tuckwell & Koziol (1992), who overlooked Malthus's distinction between unconstrained and actual populations, and applied the term 'malthusian equation' to $p = a+bt$.) If we set $K = \alpha \exp(\beta\tau)$ where $\tau = (t-t_0)$ with β taken from the population itself, we have Marx's thinking. The difference between these forms at the time of Marx's attack on Malthus was still invisible, as Malthus's is a 2-term linearization of Marx's. Later attempts to fit the US population with more data and a linear limit found that its slope had to be periodically increased (Howland 1961, Leach 1981). The lines $K_1 = 0 + 0.64\tau$ and $K_2 = -202.1 + 1.5\tau$ intersect in 1875 at 150.4 million, and work fairly well, but this is an 'eyeball' fit only, and it really needs a 3rd segment after 1980. The piecewise-linear limit has too many arbitrary parameters, unduly complex arithmetic, and does not give a consistent fit.

If β is taken from the growth of technology so that it represents our ability to control our environment, we have the ‘technagog’ equation. Written out, the technagog looks like $p' = rp\{1 - p/[\alpha \exp(\beta\tau)]\}$ and its name implies that we are ‘led by technology’. α is the initial limiting population supportable by technology at t_0.

Although the exponential equation is arguably the 2nd most important equation in science (after the linear), demographers who write well regarded books and advise on government policy have declined to review earlier versions of this paper because they felt unqualified to deal with its mathematics. Accordingly, everything one needs to know about the arithmetic of the technagog is set out in the tutorial Appendix.

Boserup (1981) discusses the incremental advance of technology, and sees it driven by population pressure. This is the reverse of the Staircase view, but it could lead to a similar ‘punctuated equilibrium’ growth of the population. Figure 1 suggests that in the US it did not. This is not a contradiction, but, I think, merely a matter of scale. Even the 1940 opening of the All-American Canal that turned a half-million acres of Imperial Valley from desert to kitchen garden with Colorado River water is a minor step compared to total US agriculture. The increments are there, but they are small, and distributed. Diffusion of new techniques also tends to smooth the effects of incremental invention. If the US is large enough and varied enough not to show effects of technological steps, the world is *a fortiori* in the same condition.

Figure 1. Technagog logistic fitted to US population data

K = carrying capacity, growth at β starting from α; *B* = biological growth at *r* starting from p_0. Filled symbols are data added since the graph was first drawn in 1969, and are thus fulfilled predictions. (A) F = spurious ‘fruitfly’ leveling predicted from 1920 to 1940 (Pearl & Reed 1920, 1930), akin to belief in the Staircase. D = the Depression. The area S is the difference between unconstrained and actual populations that Malthus investigated. The semi-log plot (B) allows easy definition of the Malthusian transition.

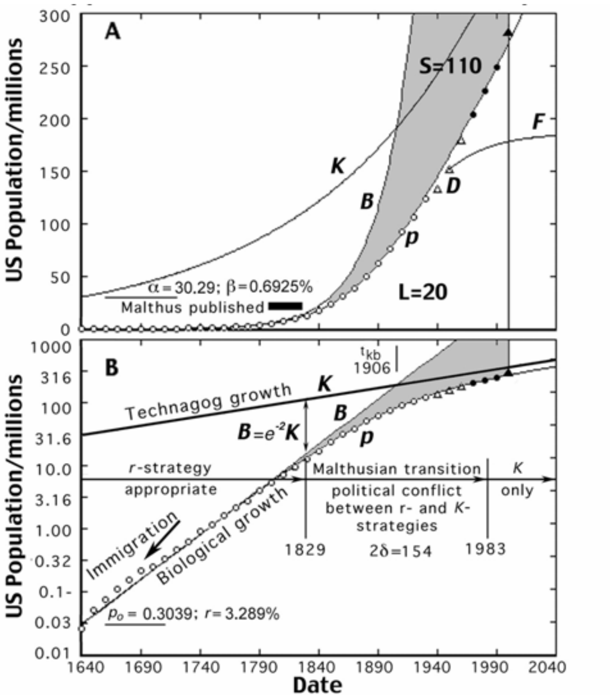


Table 1. US population data in millions [USBC]

Depression-affected counts, and the anomalous Census 2000 datum, are omitted from the curve-fitting process.

Date	***K***	***B***	Technagog	Census	PopClock
1640	30.386	0.030	0.030	0.026634	
1650	32.561	0.042	0.042	0.050368	
1660	34.893	0.059	0.059	0.075058	
1670	37.392	0.082	0.081	0.111935	
1680	40.069	0.113	0.113	0.151507	
1690	42.938	0.158	0.157	0.210372	
1700	46.013	0.219	0.218	0.250888	
1710	49.308	0.304	0.302	0.331711	
1720	52.839	0.423	0.418	0.466185	
1730	56.623	0.587	0.579	0.629445	
1740	60.677	0.816	0.802	0.905563	
1750	65.022	1.133	1.109	1.170760	
1760	69.678	1.574	1.530	1.593625	
1770	74.668	2.187	2.109	2.148076	
1780	80.014	3.038	2.899	2.780369	
1790	85.744	4.221	3.973	3.929214	
1800	91.884	5.863	5.425	5.308483	
1810	98.464	8.146	7.373	7.239881	
1820	105.514	11.316	9.963	9.638453	
1830	113.070	15.721	13.368	12.866020	
1840	121.167	21.841	17.782	17.069453	
1850	129.843	30.342	23.413	23.191876	
1860	139.141	42.153	30.465	31.443321	
1870	149.104	58.561	39.109	38.558371	
1880	159.781	81.355	49.462	50.189209	
1890	171.223	113.022	61.561	62.979766	
1900	183.484	157.016	75.353	76.212168	
1910	196.623	218.134	90.702	92.228496	
1920	210.702	303.041	107.408	106.021537	
1930	225.790	420.999	125.249	123.202624	
*1940	241.958	584.871	144.014	132.164569	
*1950	259.284	812.529	163.533	151.325798	152.271
*1960	277.851	1128.802	183.697	179.323175	180.671
1970	297.747	1568.184	204.458	203.302031	205.052
1980	319.068	2178.593	225.831	226.542199	227.225
1990	341.916	3026.601	247.877	248.709873	249.464
*2000	366.400	4204.693	270.696	281.327609	

Details of the Technagog

Figure 1 fits a technagog to 360 years of US data (Table 1). The initial point of the limiting exponential (30x10^{6}) exceeds the aboriginal population (c. 850,000 (Mooney 1928)) because it represents the supportable population using European technology. This in itself is an interesting estimate: the agriculture of 17th century Europe was able to support a population 36 times larger than the foraging and minimal agriculture of North American natives.

The 3.3% biological growth *r* doubles population in a 21-year generation, by the survival of 4 children out of perhaps 8 (as in Africa until recently), a selection rate which apparently optimized human survival in the 'arms race' between the immune system and pathogens prior to medical intervention. To help interpret Fig. 1, it appears (on the basis of limited data) that a population which exceeds the carrying capacity does not simply stop growing, but collapses into anarchy (Sen 1981, M.III.xiv). From this, we might suppose that with $B > K > p$ (as in the US since 1906):

The difference between unconstrained growth B and actual growth p is the price we pay for human culture. It can be divided into:

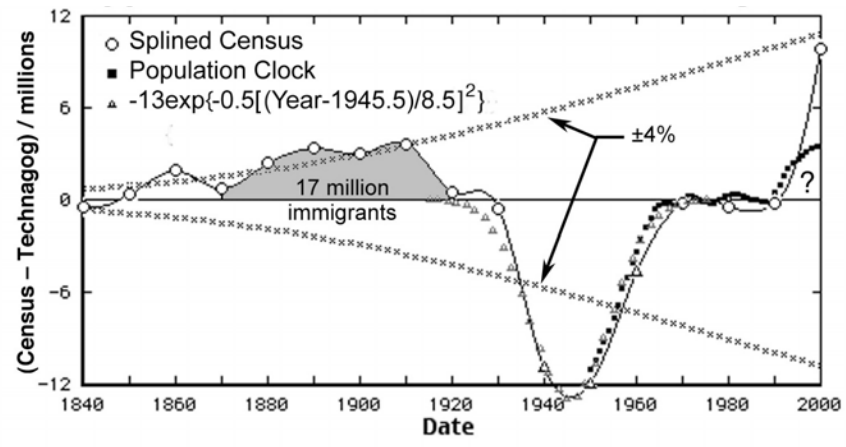
- the non-negotiable difference between B and K, which is the price we pay for the survival of civil institutions, and
- a possibly negotiable difference between K and p, which is a measure of our life style, and the price we pay for living well.

The good fit of Fig. 1 suggests that we have had a consistent image of 'the American dream' for 160 years. While it may have changed in detail, it seems to have grown in step with our mastery of technique; perhaps this can be attributed to advertizing.

Past success says nothing about the future (M.II.xiii). Population forecasts are educated guesswork depending upon the assumptions and imagination of the forecaster. (E.g.: a widely reported prediction of a population peak of 9 billion in 2070 is the central member of a suite ranging between 6 and 17 billion (Lutz 1996). The Census Bureau's 1967 projections for 2000 ranged from A = 361 to D = 282, vs the counted 281.) Line F of Fig. 1 once needed a long explanation of how such a large population could be fed (Pearl & Reed 1930). Any event comparable to the Depression could cause another deviation from the US theoretical curve, and indeed, a positive deviation may be occurring. K depends upon imports of energy and resources, for US agriculture requires increasingly scarce fossil fuel and fossil water (Gleick 1993: 56-66 & 67-79).

Figure 2. The difference (Census – technagog) for the last 160 years

The theoretical fit is better to the Population Clock than to the Census, particularly at the end of the Gaussian dip. For the first 200 years, this graph is an uninteresting straight line.



Two, or possibly three, episodes of immigration show up in the data. The first is the period 1640–1700, whose points lie above the biological-growth line in Fig. 1B, as new settlers arrive. The second shows up in Fig. 2, which examines the difference between the population and the technagog during the Malthusian transition. 17 million immigrants in 1880–1910 create a small bump (no larger than 4%, probably less than the intrinsic error of the Census). The normal expectation is that immigration will result in a step rather than a bump in the population curve, as it adds to the reproducing base. That this is not seen in Figs. 1B and 2 is a strong indication of external control. The Depression and Baby Boom are seen to be a single process, as mentioned by Cox (1970: 444). The rate of return to normal is roughly the same after both immigration and Depression excursions. Wrigley & Schofield (1981) suggest that wages drove fertility with a lag of 30 to 50 years for late medieval England, but the response of the US is much more immediate.

After 1949, the Population Clock keeps the census up to date between counts, while trying to correct for statistical anomalies. The sharpness of its break in 1965, when the population recovered from the Depression, is astonishing. The technagog value for 2000 is 271.4, which is to say that the best theoretical fit to the last 360 years underestimates the current US population, which the actual Census places at 281.3 million. This 10-million excess suggests that the effort of Census 2000 to count previously excluded groups was successful, making it statistically incompatible with previous counts. The last decade of accelerated growth in the Population Clock challenges the otherwise consistent interpretation presented here. In the absence of guiding theory, this spurt had not been recognized as anomalous by the Census Bureau. It perhaps represents both immigration, and the fertility of an unassimilated sub-population with different socioeconomic expectations. (Total US fertility fell from 2.081 in 1990 to 2.019 in 1995, but rose again to 2.075 in 1999.)

Of the four parameters of the technagog, only α is truly adjustable. The initial population p_0 and its rate of increase r are constrained by history and biology. The technological growth rate β can hardly exceed the official growth rate calculated by government economists. A challenge I leave for social scientists is to explain the parameters of the Depression Gaussian in similar detail.

Facing the Consequences

As an Anglican curate committed to moral restraint, Malthus was intellectually honest enough to face the terrible conclusion. People of less logical bent or lower emotional fortitude take refuge in psychological denial when he observes: ‘All of the children born, beyond [the supportable limit], must necessarily perish’ (M.IV.v). Nobody likes such an idea. However, the remedy is not to deny Malthus, but to recognize that the US has been following the technagog equation implacably, and unconsciously, for the last 170-odd years, while all children *conceived* beyond p have perished—*and we haven’t noticed*.

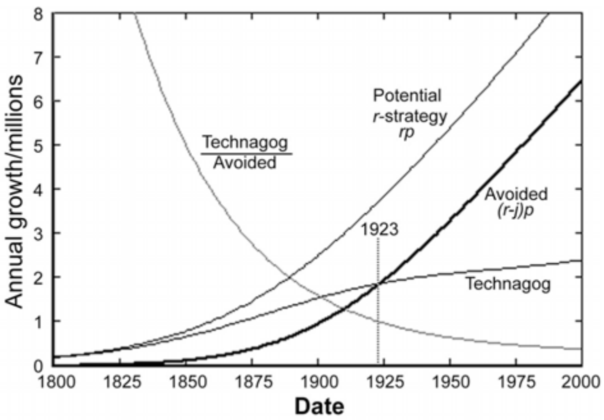
In Fig. 1A the shaded area $\mathbf{S} = \int (B-p)\mathrm{d}t = 110\mathrm{x}10^9$ represents the missing children. One cannot count individuals here, because the unit of area is the person-year. **S** contains person years possible only in the infinite fantasy world of r-strategists. $\mathbf{L} = \int p\mathrm{d}t = 20\mathrm{x}10^9$ is the person-years actually lived. One early death (or avoided birth) precludes many later, so we cannot count the deaths equivalent to **S**, but were they normal 70-year lifespans, they would number 1.6 billion—equal to the world population at the midpoint of the US Malthusian transition.

Figure 3 lets us count individuals in **S**. The ‘Potential r-strategy’ line shows growth for a given year if ‘vice’ failed and the population produced its biological quota of offspring. Had we followed this line the least of our problems would be a quadrupling of taxes to educate 4 times as many children, and a quadrupling of police and jails. The 4-fold increase assumes the infant mortality of Colonial days. With modern medicine, one might anticipate an 8-fold increase. More likely, misery would intervene in ways we would rather not think about.

As the number of avoided births rises, the ratio between the actual and the potential steadily decreases, as shown by the line Technagog/Avoided in Fig. 3. This reached unity in 1923 without attracting much attention. This brings us to the crux. The US has only rarely resorted to misery in its Malthusian transition, and then only for marginal groups. As Malthus realized, moral restraint has never been effective (M.I.xiv), because it goes against the grain of human nature, and it is useless in the hands of the world's arbiters of morality, whose Neolithic Scriptures encourage, or even demand, an *r*-strategy (Reynolds & Tanner 1983). The key lies in the previous paragraphs: **one birth avoided by 'vice' prevents one death by misery**. This will be a difficult fact for many to accept because cause and effect are separated in time and space. Nevertheless, one failed contraception, or one prevented abortion, any time since 1840, has resulted in one pre-reproductive death by misery somewhere else in the country. Otherwise Fig. 1 would not have its characteristic shape. Cause and effect are not only separated, but ambiguous here. The Malthusian transition was very directly the effect of 'vice'. 'Vice' is an individual voluntary choice; misery is collective and involuntary—yet both, by the Malthusian hypothesis, lead to the same result. How can this be unless the technagog equation is somehow the 'cause'? This ambiguity places the technagog beside the mysterious 'invisible hand' of Adam Smith, where millions of independent decisions made in isolation lead to an efficient market which supplies goods at minimum cost. With 'vice', millions

Figure 3. Details of the Malthusian transition

The 'Potential *r*-strategy' line shows the population increase in any year if 'vice' failed. The line 'Avoided' shows the children who would have died by misery had they not been prevented by 'vice'. The ratio 'Technagog/Avoided' dropped to unity by 1923, although noise in the real system (Fig. 4B) obscures this. Awareness of this ratio has risen as the ratio has fallen below 1, leading to protests despite the non-political nature of Malthusian control.



of decisions, many unconscious, irrational, or accommodating wholly irrelevant concerns, nevertheless led to the accord with theory of Fig. 1. Do the invisible hand and technagog foreshadow the quantitative 'psychohistory' of Isaac Asimov's *Foundation* novels?

Those who oppose family planning face the same cruel dilemma as Malthus: *r*-strategists always have predators. Humans have none save pathogens, parasites—and other humans. If surplus children are born, surplus children will die, and the definition of 'surplus' is not ours to make. The 20th century—our bloodiest yet (Rummel 1996) because most populous—brought new mechanisms for family planning and population control. 'Vice' became more sophisticated and reliable. Inadvertent new population miseries—which affect us all in unquantified ways—include pollution, environmental estrogens (Colborn &a 1996, Cadbury 1997) climate change and violent weather, and drug-resistant and emergent diseases (Lederberg &a 1992). Intentional new (or newly refined) miseries include democide, child soldiers with Kalashnikovs (Boothby & Knudsen 2000), organized crime, hard drugs, terrorism, uranium and depleted-uranium munitions, land mines, inexpensive firearms (the number of manufacturers has tripled in 25 years), and incarceration in the US (Gibbs 1995) and many non-democratic regimes. To the 200 million killed by their own governments in the 20th century we must add 50 million displaced persons (Rekacewicz 2001), all without homes, schools, or health care. These miseries may be negative feedback, like the behavior of Calhoun's crowded rats (1962). Moore (1999) suggests that the behavior Calhoun called 'maladaptive social pathology' was in fact adaptive and economic (in terms of energy expended to attain goals). This, it would seem, brings the rats' infanticide, cannibalism, and sexual displacement activity into even closer congruence with human behavior under stress. Perhaps, despite superficial involvement of the usual excuses for mass murder (religion, politics, megalomania, &c), a major unconscious drive behind the misery of the 20th century was Malthusian competition for *Lebensraum* (Ratzel 1897, M.I.viii). Received opinion holds that 'population growth alone is rarely the cause of violent conflict' (Brown &a 1999: 97, 99), but the same source immediately recognized 4 exceptions: Karachi, Chiapas, Ningxia, and Rwanda.

Lacking a coherent world-population policy, we tolerate misery as a substitute, yet it seems that an intelligent and moral species would find a less painful approach. The shaded area of Fig. 1 offers hope that the classical economists' goal of alleviating human misery is achievable ('I have known Adam Smith ..., Ricardo, and Malthus .... Is it not something to say for a science that its 3 great masters were about the 3 best men I ever knew?' Sir James Mackintosh; M.Intro. p. vi; see also M.IV.passim). This is testable, if not by retrospective analysis, then by Pimentel's method (below) of a century of world birth control. If, as Malthus observed of moral restraint, 'diffusing knowledge and advice on the subject would be favorable to good government, social order, and peace' (M.IV.xiv), then diffusing the means for birth control would be even more favorable. Alternatively, we can fail to provide this and monitor the increase in bloodshed.

Over the last 200,000 years, those who saw overpopulation as a problem, or otherwise failed to maximize their own offspring, succeeded only in removing such responsible traits from the gene pool (Hardin 1968). DNA analysis indicates that all humans carry the mitochondria of a single woman (Cann &a 1987) and all males the Y chromosome of one or two men (Hammer 1995). The significance to population studies is that the genetic fecundity (Gagneux &a 1999) of these our ancestors was sufficient to out-breed and exterminate all other lineages. (Jobling &a (1998) shows a selective advantage to Y-chromosome haplotypes.) I suggest that our 40-year flirtation with the Staircase can only mean that we are genetically protected against believing that overpopulation is a problem.

This is hardly a novel observation. At the 1968 Princeton Conference on Population (sponsored by the Interdisciplinary Communications Program, the New York Academy of Sciences, and the Smithsonian Institution), Gordon Rattray Taylor raised the issue of the optimum population.

'Demographers are embarrassed when the question of optimum population is raised.... [T]here was a shocked silence, after which the chairman ruled that that subject be postponed to the end of the meeting. It was never, in the end, discussed.' (Taylor 1970: 225).

Taylor declined to suggest what the optimum number might be, but ended his chapter with a very practical and highly ethical limit proposed by Athelstan Spilhaus (who was, be it noted, not a demographer but an oceanographer): 'When we can treat all existing persons as human, it will be time enough to think about having more'. Today I would add the qualification that it is necessary to do this in an environmentally sustainable manner.

A Projection

The fit of the technagog to the US population offers encouraging support for the Maltho-Marxian hypothesis. If the technagog works for the US, does it perhaps work for the world too? If it does, we may not like the result. 'The fifth revolution will come when we have spent the stores of coal and oil that have been accumulating in the earth during hundreds of millions of years.... This change may justly be called a revolution, but it differs from all the preceding ones in that there is no likelihood of its leading to increases of population, but even perhaps to the reverse' (C. G. Darwin 1952). For a recent attack on this view, executed with erudition and panache, see Salter (2005). Unfortunately, Salter advanced no data at all in support of his position. The combination of a Verhulst logistic and a Gaussian supports Darwin's realistic outlook.

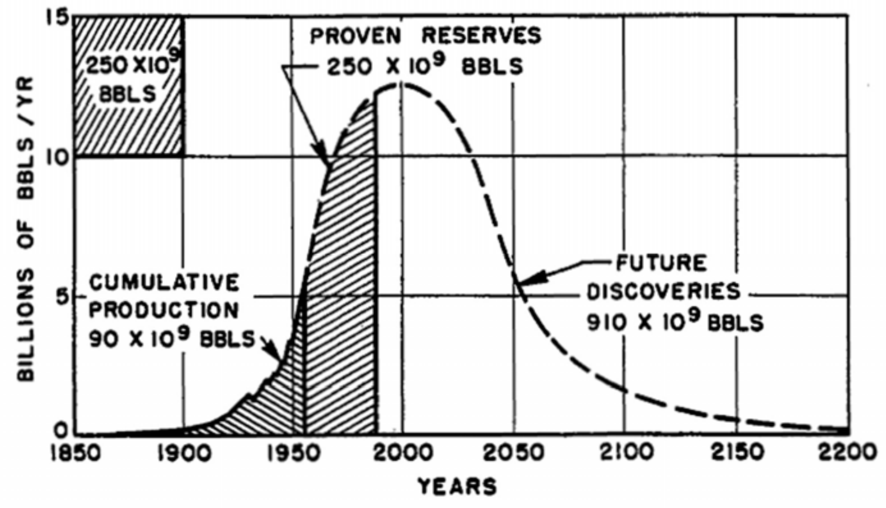
Short-term projections for world population range from the 8-10 billion of the UNPD [2004] down to Pimentel's 2-3 billion (Pimentel &a 1994). The difference lies in the much abused word 'sustainable', and the life style which is to be sustained. Pimentel, for instance, specifically cites 'half the US standard of living' as sustainable at a population of 2 billion. Although there are small-scale experiments which indicate that we know what needs to be done, *nothing* we are doing today qualifies as sustainable, This includes agriculture, animal husbandry, biodiversity protection, fishing, forestry, genetic modification of crops, power consumption, sewage disposal, transport, urban design, water use, &c. '[B]ased on past experience, ... these urgent issues concerning human carrying capacity of the world may not be addressed until the situation becomes intolerable or, possibly, irreversible', say Pimentel &a (1999).

If we apply the Maltho-Marxian hypothesis to the world population, the fruitfly logistic is the most plausible, at least until we demonstrate that sustainable solutions can be found which are themselves amenable to technological advance. This is a realm often explored in science fiction, and offers many ideas that might work. Example: At mid-century Pohl and Kornbluth's *The Space Merchants* (1952) introduced the idea of meat production by industrial tissue-culture techniques. New Harvests, founded in 2004 to follow up this idea, is planning to build a 2-liter rotating reactor (http://www.new-harvest.org/default.php). This may be a giant step forward, but many good ideas fail when total environmental costs are included. An important point is that no matter how many clever ideas we have, the window of opportunity to deploy them in time to escape a population collapse is narrowing.

In 1956 M. King Hubbert, Shell Oil's Chief Consultant (General Geology), estimated that the date of peak oil production would be 2000 (Hubbert 1956), as shown in Fig. 4. Realistic current estimates run from 2003 to 2020, with the proviso that it will take five years to be sure that we are seeing the real peak and not noise. Unrealistic estimates extend to 'When will we run out of oil? Never!' (Simon 1981) which is true only in the sense that there will always be some petroleum remaining below ground, too expensive to extract.

Figure 4. M. King Hubbert's 1956 projection of peak world oil production

We can hardly claim to be surprised by the idea of 'peak oil'.

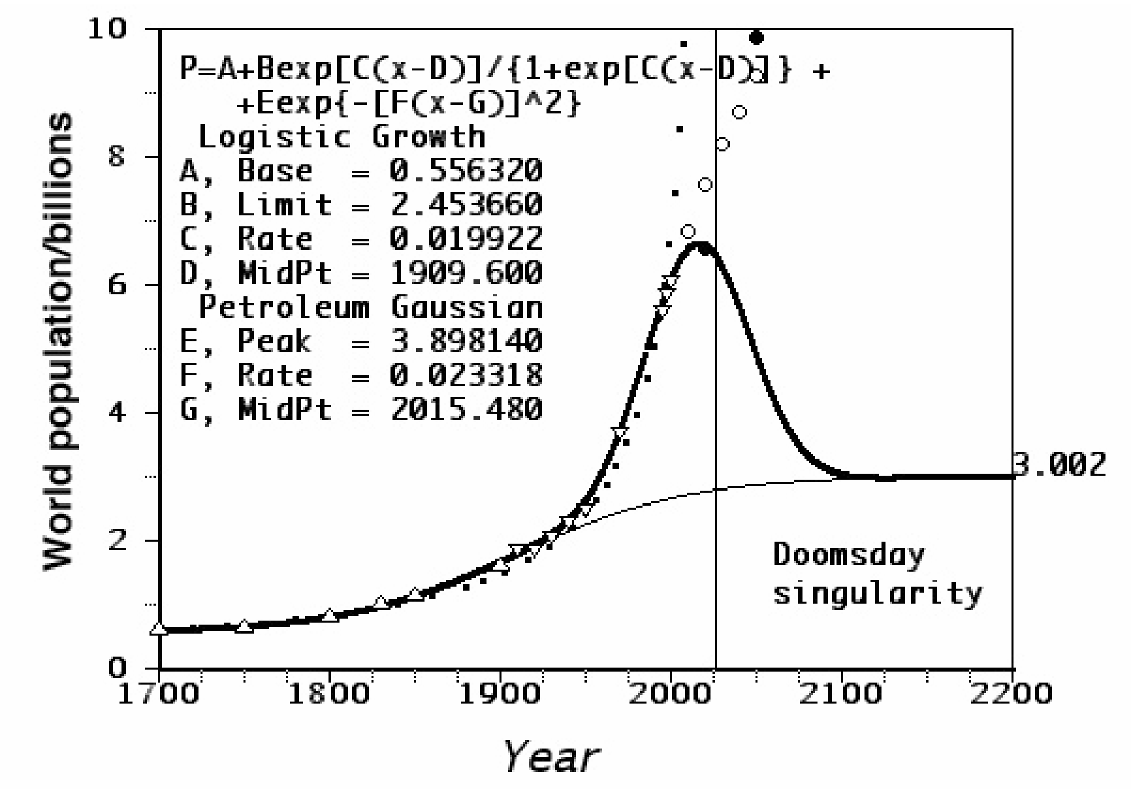


Combining Pimentel's estimate and Hubbert's oil exhaustion with the Maltho-Marxian hypothesis leads to Fig. 5, where circles are various UN projections. 'Hubbert' curves are not Gaussians (Laherrère 2000), but the difference is small. The absence of information about how we might reach a sustainable population of 2-3 billion means that there is no preferred way of describing the downside of the hump. A simple Gaussian will do.

What is most striking about Fig. 5 is that none of the Gaussian parameters are specified in advance, yet for any final population between 2 and 3 billion, the Gaussian is nearly identical, and agrees with current ideas about the date of peak oil. I repeat, this feature is the *output* of the fitting process, not input. All correlation coefficients in this range are better than 0.999, which at least

Figure 5. The world population (data from various UN publications) fitted to a logistic using Pimentel &a's (1994) high estimate of the supportable population plus a Gaussian excursion supported by the fossil energy of petroleum.

This approach combines the law which governs all animal populations with the uniquely human ability to support a supra-environmental population using the energy from fossil hydrocarbons.



suggests that the concept of a population reduction—however unpopular—is mathematically robust. (There are too few data points for bootstrapping or confidence intervals, and earlier population estimates are too inaccurate to be helpful.)

The infamous Doomsday paper (Von Foerster &a 1961) showed that *if* the world population maintained the behavior of the preceding 2000 years, it would go infinite on Friday, 13 November 2026 and squeeze itself to death. This equation is shown by the dotted line in Fig. 5, asymptotic to the vertical Doomsday Singularity. The authors properly pointed out that in physics, the appearance of such a singularity means that behavior will change. (Melting points are singularities. The behavior on either side of the singularity is regular and predictable, but the rules have changed and the behavior is different.)

Unlike most projections, which are exponential, the best fit found by von Foerster was a hyperbolic equation developed to follow the growth of tumors. Gregg (1955), Forencich (1992/93), and Hern (1999) have also noticed this feature. The hyperbola implies an additional contribution to population growth beyond the proportionality to the size of the reproducing population expressed by the exponential equation. The tumor manages this by inducing angiogenesis, compelling the surrounding tissue to provide it with an extra blood supply comparable to that seen during the healing of wounds. Humans manage this by forming a coalition against nature, inventing technology that increases the supportable population, just as in the technagog. Despite its manifest improbability, the Doomsday hyperbola is noted for being ‘the most accurate predictor of real population growth for almost two generations’ (Smith & Moore 2001).

The combination of a logistic and a Gaussian nicely mimics the rise of the hyperbola until it gets close to the singularity. It seems to provide everything needed: von Foerster's 'coalition', the irresistible force needed to change behavior and avoid the singularity, and perhaps a projection of the actual population. For much of the world, the effects of oil shortage will be as unexpected as the island crises mentioned above.

The Doomsday paper gave us a 65-year lead time. Most published comments failed to grasp its central point: the demographic behavior of 2 millennia *will* change by 2026, willy-nilly. At a 51-year lead time (Serrin 1975), and again at a 39-year lead time (Umpleby 1987), the population was still ahead of the Doomsday projection. We had done nothing. This is presumably the result of denial: ‘The human population *cannot* decline.’ Malthus observed that the ‘first grand objection’ (M. Appendix) to a limit was Genesis 1:28, the infamous ‘multiply-fill-subdue-master’ verse singled out by Lynn White, Jr. (1967) as the root of our ecological problems. There is no reason to think that denial has decreased since Malthus wrote. We have experienced population crashes before, both from vulcanism like the Young Toba Tuff of 75,000 BP (Ambrose 1998) and from the Black Plague of the 14th century (Herlihy 1997). The problems described in Mesopotamian epics arose from overpopulation and were cured by Malthusian misery (Moran 1971, Kilmer 1972). We should actually welcome Fig. 5 as describing a plausible (indeed, unavoidable) mechanism for forestalling Doomsday. It means that the world no longer faces death by compression. This detail may not matter to those who exceed the limit, and times will be no doubt be interesting for the survivors, but at least survivors are now predicted.

It is obvious that the projections of the UN and Pimentel cannot both be correct. The UN estimate is a political compromise based on demographics, the assumption of business as usual, an infinite oil supply, and on the difficulty of group agreement on a paradigm switch which goes counter to everything believed from tradition, history, religion, and aspirations. The precautionary principle, unpopular though it is, suggests that we would do well to consider immediate implementation of palliative measures. The consistent advice of the economists—‘Yes, but wait 50 years until we are all rich before spending all that money’—is not apt to ameliorate the situation.

There are many approaches we might take. Pimentel’s own suggestion reveals him as a kindly inhabitant of an ivory tower: ‘The adjustment of the world population from 6 billion to 2 billion could be made over approximately a century *if* the majority of the people of the world agree that protecting human health and welfare is vital, and all are willing to work to provide a stable quality of life for ourselves and our children’ (Pimentel &a 1999) (emphasis added).

To be fair, this was written before 9/11 and its demonstration that not everyone agrees that ‘human health and welfare’ are the first priority. Also, those who believe themselves divinely commanded to out-breed the infidels cannot in conscience limit their own population.

We might have used the half-century warning about peak oil production to plan for it. The US, user of 25% of the world’s oil, took until 2005 to begin the sort of development that Denmark, user of 0.25% of the world’s oil, began in 1973, after the first oil-price shock, so that 42% of Denmark’s domestic heating is now supplied by waste or biofuel (DEA 2005). There may still be time to build the infrastructure for renewable sources of electrical energy, but architects’ plans for integrated wind-turbines in buildings to supply a token 10-20% of the buildings’ power needs are admired but are not being built. Biofuels (including charcoal, currently a non-sustainable use) account for 14% of the world’s primary energy production, yet not all biofuel projects are helpful. Biodiesel from Jatropha trees in India may reverse desertification (Bhagat & Bhagat 2005), but Patzek (2004) describes many biofuel schemes as ‘fundamentally flawed’ and finds that producing *sustainable* ethanol from maize requires 2.4 times as much energy as the ethanol releases to maintain soil fertility. (US agribusiness has never operated on a sustainable basis. We have lost half of our topsoil in 200 years, and used fossil water faster than aquifers can recharge, which compacts them and reduces their capacity.) The *Atlantic Monthly* had a sober assessment of the coming century of cold war with China which completely ignored the oil problem (Kaplan 2005). China is already buying Canadian oil which the US felt it had a moral claim on. The Bush administrations’ wars in the Middle East suggest that there is awareness in Washington of a problem with oil in the future, but the approach seems counterproductive. What we need are sustainable alternatives for the many uses of oil.

Figure 5 is probably the optimistic default outcome. If you don’t like it, priorities need to be set to avoid it. Graham Zabel (2000, Fig. 3) has drawn the rising portion of this figure in more detail, dividing it into biomass-, coal-, oil-, and natural-gas-supported sub-populations. His graphs end in the year 2000 so that he need not depict a falling total population, but he does show the sub-populations falling, and suggests, ‘*Oil Population* may decline more quickly than most people anticipate’.

Conclusions

If the technagog logistic be merely a pocket-calculator replacement for Census tables, it nevertheless fits 360 years of US population data with simple, meaningful, parameters. It suggests useful distinctions among the transitions of Fig. 6 (growth-rate, biological Gaussian, and a possible demographic transition marked by a minimum in the annual increment), none of which is obvious in the raw data. It reveals that the ‘baby boom’ exactly compensated the birth deficit of the Depression, a finding not apparent from other approaches. One critic claimed that demographers had indeed predicted the baby boom. Perhaps someone did: it is hard to prove a negative. But if so there was a major failure of communication, for in 1951 the received opinion was ‘Between 1940 and 1950 the US experienced the largest numerical population increase in history.... This ... was not anticipated: the 1950 total ... was about 7 million above the highest prediction made by population experts...’ (Anonymous 2001).

If, on the other hand, the technagog be an explanatory equation, it shows that after 1829 the population responded to economic conditions by regulating itself so that it did not too closely approach the carrying capacity, just as Malthus predicted. The precision of fit of data to theory, and the fit of the equation to Malthus's verbal description, are remarkable. This point needs expansion, because other equations have fitted this or that population well for certain periods, so there is nothing remarkable about a good fit. Yet of the 4 parameters of the technagog, only α is unconstrained by auxiliary data. β, the growth rate of the supportable population, is about 1/3 that estimated by governments and economists, and this in itself is interesting. I suggest that the technagog estimate is a more reliable measure of real *economic* growth, as it is neither running for election nor trying to jawbone the stock market. Perhaps 'official' estimates ignore transaction costs, neglected maintenance, and unbuilt infrastructure.

Four major sociological data sets (world fertility, life expectancy, under-5 mortality. and GDP) correlate so well with years of education that they are used to set billion-dollar budgets. Yet the correlation coefficients are only 0.71, 0.82, 0.73, and 0.78 respectively (UN 2004, Summary figures 1-4). The correlation coefficient for the technagog is 1.00000. Those to whom such measures are meaningful might check the relevant paragraph of the Appendix for additional data on the technagog fit.

I suggest that one of the world's major problems is that too few politicians have an intuitive understanding of the exponential equation, and are always caught unawares when their linear extrapolations land them (and us) in trouble. The Appendix may look daunting, because by editorial request it is compressed and does not explain its equations line by line. Yet it deals only with the calculus of the exponential—the master equation of growing things—which should be accessible to any college freshman and required of anyone who aspires to public office. Its bottom line is that exponential growth on the planet cannot long continue; the environment always imposes a limit; the only question is the nature of the limit.

I suggest that the school boards, teacher-training colleges, law schools, and seminaries of the US have not done an adequate job of preparing their pupils. It is impossible to think rationally about population without understanding the exponential function. If we want domestic tranquility, politicians—and before them, preachers and a majority of voters—need this basic nonlinear tool. This is not a new idea, but an extension of one known since Classical Greece: Mastery of geometry is a precondition of logical thinking.

Philosophy of Science

There are 2 very different questions to be sketched under this rubric. The first is the matter of testing a hypothesis which has only a single good data set. There are many populations which do not follow the technagog, or anything else. All I can suggest is that other species nearly universally follow logistic growth. The limit is environmental and changes as the environment changes (i.e.: K is not always a constant.) We continually assert that technology grows exponentially and that we depend upon it. This make the technagog the simplest plausible hypothesis for the human population. The 'Depression Gaussian' of the US is a blessedly simple instance of a consistent departure from the smooth technagog.

The other question relates to our benign perception of population growth, and is asked by those who study bias in reasoning: 'Why [is] apparent competence in logical and statistical reasoning exhibited under one set of circumstances'—particularly by the readers of *Scientific American*—'so frequently absent in others[?]' [Evans, 1994: 7]. Why, in other words, did the erroneous Staircase survive so long despite repeated attempts at rational correction? Tabular data to refute the Staircase are given 2 pages after the illustration in Livi-Bacci's book: Why did he—Professor of Statistics and President of the International Union for the Scientific Study of Population—not notice this? Why is the optimum population not a topic for active discussion? Is it truly the case that DNA blinds most of us to the undesirable effects of overpopulation?

Afterthought

I will follow that last question with a wild speculation in the hope of interesting a competent investigator. Genetics might not be the only source of our reluctance to consider an optimum population. Well adapted parasites not only evade immune systems, but alter the behavior of their hosts in aid of their own survival. Rats infected with toxoplasmosis lose all fear of cats, the reproductive milieu for toxoplasmids (Berdoy &a 2000). Tachinid fly larvae change the diet of their host caterpillars to increase the *caterpillars*' survival (Karban & English-Loeb 1997). Tapeworms persuade sticklebacks (Svensson & Woodhouse 2001) and roach (Loot &a 2002) to swim slowly at the surface so birds can eat them. *Acanthocephalus dirus* persuades its intermediate isopod host to engage in conspicuous 'eat me' behavior to attract its reproductive host (fish) (Beeson &a 2005). A startling case is the lancet fluke, *Dicrocoelium dendriticum*, which moves through the species sequence cow-snail-ant-cow in its life cycle. In the ant phase, it drives ants to leave the nest at night, climb a grass stem, and cling to its tip in the hope of being eaten by a cow. Exposure to the heat of the day being detrimental to the parasite, at sunrise the ant is freed to climb down and seek its nest (Zimmer 2000). Adds Zimmer, 'we, too, are collections of cells that work together, kept harmonized by chemical signals. If an organism can control those signals—an organism like a parasite—then it can control us.'

E. coli has had a long and rewarding relationship with animal guts, where it is a commensal rather than a parasite (thus extremely well adapted). There has never been an adult human free of *E. coli*, so we have no idea how one might behave. But one new human gut means 10^7 more *E. coli*, so they have a vested interest in our reproduction. Is it possible that they contribute chemically to our inability to recognize that overpopulation is a problem? A closely parallel example is the production of a rodent growth-hormone mimic by the cestode tapeworm *Spirometra mansonoides* (Moore 2002: 153; Fig. 5.9). Are we, perhaps, thinking with the contents of our large intestines when it comes to population problems?

Acknowledgements

The first stages of this work were supported by a John Simon Guggenheim Memorial Fellowship. Michael McLaughlin, creator of Regress+ (http://www.geocities.com/~mikemclaughlin/software/Regress_plus.html) provided an excellent free curve-fitting program and cogent advice about using it to best advantage. I thank Joel Cohen, Ward Elliott, the late Garrett Hardin, Georgia Lee, Ronald Lee, Nicholas Morley, and Stuart Umpleby for comments on early drafts, and Ben Bolker for guidance on parasites.

Appendix: Arithmetic of the Technagog

Whatever simple equations may miss by parametrizing sociological and economic influences, they compensate by expanding the universe of discourse with the additional useful, but experimentally inaccessible, parameters of Fig. 6.

Derivatives and integral of the technagog

$B = p_0 \exp(r\tau)$, $K = \alpha \exp(\mathrm{b}\tau)$, $\tau = t - t_0$, $t_0 = 1640$; B and K as in Fig. 1.

$K' = (\partial K/\partial t) = \beta\alpha \exp(\beta\tau) = \beta K$

$p = \int p'\,\mathrm{d}t = B/[1 + B/RK]$, $R = (r - \beta)/r$ (A1)

$p' = (\partial p/\partial t) = rp(1 - p/K)$; or pq (A2)

$p'' = (\partial p'/\partial t) = r[p' - (2pp' - p^2\beta)/K]$; or $p'q + pq'$

$p''' = r\{p'' + [4\beta pp' - (p\beta)^2 - 2(pp'' + p'^2)]/K\}$; or $p''q + 2p'q' + pq''$

$q = r(1 - p/K) = p'/p$

$q' = (\partial/\partial t)(p'/p) = (pp'' - p'^2)/p^2 = p''/p - q^2$; or $(rp/K)(\beta - q)$

$q'' = p'''/p - p'p''/p^2 - 2qq'$; or $(r/K)[p'(\beta - q) + p(\beta q - \beta^2 - q')]$

$\partial p/\partial p_0 = (p/p_0)[1 - (p/RK)]$

$\partial p/\partial r = p\tau + (p^2/RK)[\beta/r(r - \beta) - \tau]$

$\partial p/\partial \alpha = p^2/\alpha RK$

$\partial p/\partial \beta = p^2[\tau - 1/(r - \beta)]/RK$

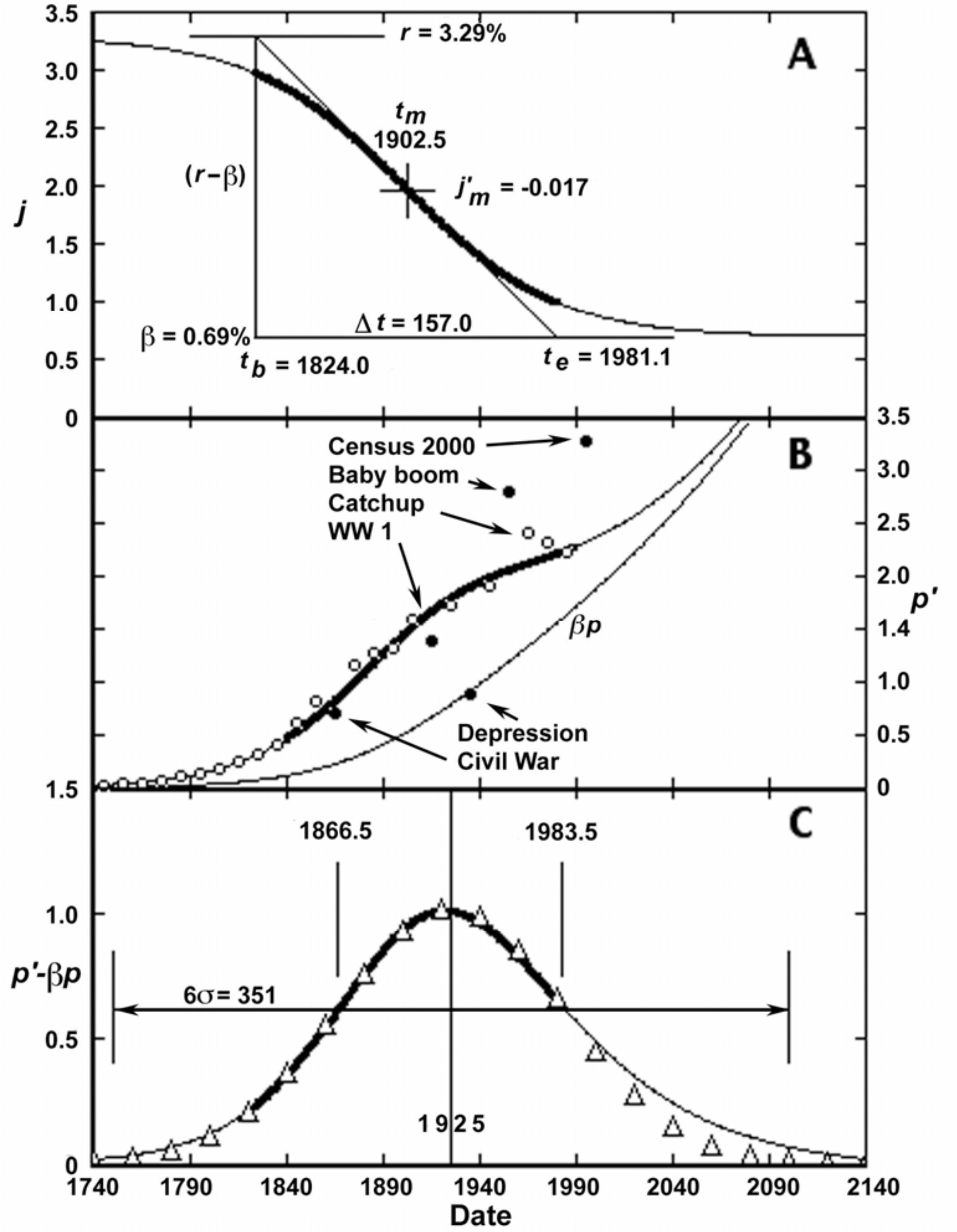
Approximate parameter values for the US are: $\alpha = 30\text{x}10^6$, $\beta = 0.69\%$, $p_0 = 0.030\text{x}10^6$, $r = 3.3\%$, $R = 7/9$. The exact values given in Fig. 1 were obtained from Michael McLaughlin's Regress+; the minimization criterion was least square deviation, the weighting function for point p_i was $1/\sqrt{p_i}$, and the 4 points marked by triangles (last, and 3 during the Depression, which for historical reasons do not lie on the theoretical line), were excluded from the fitting process.

Figure 6. Population transitions

(A) Growth-rate transition Δ *t* from linearizing $q = p'/p$. The Staircase expectation is that this curve will return to zero, but even Livi-Bacci's 'European' model behaves as shown here (1989: Fig. 4.2), although he puts no numbers on his axes.

(B) *p'* reveals disturbances such as World War I. The heavy portion of the curve is Δ *t* from (A). I suggest that *p'* departs from and re-approaches its asymptote, β*p*, so that what returns to zero is not the growth rate but the 'biological Gaussian'.

(C) Biological growth, after subtracting technagog growth, is well fitted by a Gaussian (the triangles). The Gaussian is fitted only at the marked ±1σ years**.**



Linearizing the annual fractional growth curve

Fig. 6A shows the conventional way to define the onset of the 1-sided demographic transition. The slope q_m' of q evaluated at the zero of q'' at t_m gives the triangle of Fig. 6A, where $q_m' = -(r - \beta)/\Delta t$ to obtain $t_b = t_m - \Delta t/2$ for the start of the transition.

Another estimate of the midpoint of the growth transition

We take as midpoint t_{kb} of the growth transition the intersection of lines K and B in Fig. 1B. When K and B are equal, we have

$$K(t_{kb}) = \alpha \exp(\beta t_{kb}) = B(t_{kb}) = p_0 \exp(r t_k) \qquad \text{(A3)}$$

and thus

$$t_{kb} = t_0 + \ln(\alpha/p_0)/(r - \beta)$$

A small difference between t_m and t_{kb} can be ignored.

Length of the growth-rate transition without 2nd derivatives

Fig. 1B suggests that the growth-rate transition occurs while B lies between $e^{\pm 2}K$. The ends of this interval are δ from their intersection. Then at $t_\delta = (t_{kb} \pm \delta)$ we have

$$B(t_\delta)/K(t_\delta) = [Ce^{\delta r}/Ce^{\delta\beta}]^{\pm 1} = ln\ 2$$

whence

$$\delta(r - \beta) = 2$$

and $2\delta = \Delta t = 4/(r - \beta) = 154$ years.

After the transition

The growth rate will asymptotically approach the technological growth β, or

$$q = r(1 - p/K) \rightarrow \beta$$

whence $p \rightarrow (1 - \beta/r)K$ so that the population parallels the limit as in Fig. 6B, displaced by the constant factor β/r. At this time the governing equation reduces to

$$p = (1 - \beta/r)\alpha \exp(\beta\tau) \qquad \text{(A4)}$$

$$p' = \beta(1 - \beta/r)\alpha \exp(\beta\tau) = \beta p$$

Was it a coincidence that the minimum growth rate of the Depression was also βp?

Statistical measures

The quality of the technagog fit is more characteristic of physical sciences than of biology. Add to this the facts that the theory was formulated 200 years ago, and that for the last 40 years its numerical predictions have been verified, and one might want statisticians to answer the basic question: Can we reject the hypothesis that the technagog is a valid description of US population history? Here are the numbers returned by Regress+ (N.B.: Curve-fitting algorithms work most safely with numbers near 1, so all population numbers are in millions.):

R-squared = 1.00000

Average Deviation = 8.72929e-02

Total-sum-of-squares = 1.76207e+05

Standard-error-of-estimate = 1.05670e-01

Results from non-parametric bootstrap of 5000 samples, using bias-correction with acceleration:

Covariance Matrix:

p_0	r	α	β	
4.25279e-06	-8.01870e-07	5.92859e-03	-5.50026e-07	p_0
	1.75017e-07	-1.23928e-03	1.31981e-07	r
		1.30408e+01	-1.24485e-03	α
			1.33404e-07	β

Parameter	Best Fit	Bootstrap mean	95% Confidence limits:
p_0	3.03929e-02	2.93100e-02	[2.80647e-02, 3.48123e-02]
r	3.28884e-02	3.30850e-02	[3.21185e-02, 3.34137e-02]
α	3.02938e+01	2.94950e+01	[2.53818e+01, 3.73081e+01]
β	6.92487e-03	7.02399e-03	[6.29475e-03, 7.46851e-03]

References

Abramowitz, M. and I. A. Stegun (1965). Handbook of Mathematical Functions. New York: Dover.

Ahlburg, DA (1995). “Simple versus complex models: evaluation, accuracy and combining.” Mathematical Population Studies, 5(3): 281-290.

Ambrose, S. (1998). “Late Pleistocene human population bottlenecks, volcanic winter, and differentiation of modern humans.” J. of Human Evolution 34: 623-651.

Anonymous (2001). “50, 100, & 150 Years Ago.” Scientific American April, p 8. (UK pagination.)

Bahn, P.G. and Flenley, J. (1992). Easter Island, Earth Island. London: Thames and Hudson.

Beeson, E.R., Frye, K.L., and Sparkes, T. C. (2005). “Field Evidence of Parasite Host Modification in Freshwater Streams.” 25th Midwest Ecology and Evolution Conference, Southern Illinois University, Carbondale IL 11-13 March 2005/Poster

Berdoy, Manuel, Joanne Webster and David Macdonald (2000). “Fatal Attraction in Rats Infected with Toxoplasma gondii.” Proc. Roy. Soc. London B 267: 1591–1594.

Bhagat, Brook and Gaurav (2005). “Hope In Jatropha: India Gives Biofuels a Chance to Grow.” http://www.ecoworld.org/Home/Articles2.cfm?TID=353

Boothby, N. G. and Knudsen, C. M. (2000). “Children of the Gun.” Sci. Amer. 282: 40-45.

Boserup, Ester (1981). Population and Technology, Oxford: B. Blackwell.

Brown, L. R. Gardner, G. and Halwell, B. (1999). Beyond Malthus. New York: Norton.

Campbell, C.J. (1999). “The Imminent Peak Of World Oil Production.” Presentation to a House of Commons All-Party Committee on 7 July 1999.

Cadbury, D. (1997). The Feminization of Nature. London: Penguin.

Calhoun, J. B. (1962). “Population density and social pathology.” Sci. Amer. 206: 139-148.

Cann, R. L., Stoneking, M. and Wilson, A. C. (1987). “Mitochondrial DNA and human evolution.” Nature 325: 31-36.

Cohen, J. E. (1995). How many people can the Earth support? New York: Norton.

Cohen, M. (1977). The Food Crisis in Prehistory: Overpopulation and the Origin of Agriculture. New Haven: Yale University Press.

Colborn, T., Dumanoski, D. and Myers, J. P. (1996). Our Stolen Future. London: Penguin.

Cox, P. R. (1970). Demography, 4th ed. Cambridge University Press.

Darwin, Sir Charles Galton (1952). The Next Million Years. London: R. Hart-Davis.

DEA, Danish Energy Authority (2005). “Basic facts on heat supply in Denmark.” http://www.ens.dk/sw20278.asp.

Deevey, E. S., Jr (1960). “The human population.” Sci. Amer. 203(#3): 194-204, 278.

Durie, M. J. (2003). “Witness Statement.” Victorian Civil and Administrative Tribunal, Anti-Discrimination List No. A392/2002.

Ehrlich, Paul and Anne Ehrlich (1970). Population, Environment, Resources. San Francisco: W.H. Freeman.

Evans, J. StB. T. (1994). Bias in Human Reasoning: Causes and Consequences. Hove, UK: Erlbaum.

Ferdon, E. N. (1981). Early Tahiti: As the Explorers Saw it: 1767-1797. Tucson: University of Arizona Press.

Forencich, Frank. (1992/93). “Homo Carcinomicus: A Look at Planetary Oncology.” Wild Earth 2(4): 72-74.

Fremlin, J. H. (1964). “How many people can the world support?.” New Scientist #415: 285-287.

Gagneux, P, Wills, C, Gerloff, U, Tautz, D, Morin, PA, Boesch, C, Fruth, B, Hohmann, G, Ryder, OA and Woodruff, DS (1999). “Mitochondrial sequences show diverse evolutionary histories of African hominoids.” Proc. Natl. Acad. Sci. USA 96: 5077–5082.

Gibbs, W. W. (1995). “Seeking the Criminal Element.” Sci. Amer. 273 (Nov.): 77-83 (UK pagination).

Gleick. P. H. (ed) (1993). Water in Crisis: A Guide to the World's Fresh Water Resources. New York: Oxford U. Press.

Gregg, A. (1955). "A Medical Aspect of the Population Problem." Science 121: 681-682.

Hammer, M. F. (1995). "A recent common ancestry for human Y chromosomes." Nature 378: 376-378.

Hardin, G. (1968). "The Tragedy of the Commons." Science 162: 1243-1248.

Hardin, G. (1999). The Social Contract 9(#3): 139-145.

Herlihy, David. (1997). The Black Death and the Transformation of the West. Cambridge, MA: Harvard University Press.

Hern, W. M. (1999). "Why Are There So Many of Us? Description and Diagnosis of a Planetary Ecopathological Process." Population and Environment 21(#1): 59-80.

Howland, W. E. (1961). "Letter." Science 133: 939.

Hubbert, M. King (1956). "Nuclear Energy and the Fossil Fuels." Shell Development Company Publication #95, 'to be published in Drilling and Production Practice 1956'.

Jobling, M. A., Williams, G., Schiebel, K., Pandya, A., McElreavey, K., Salas, L., Rappold, G. A., Affara, N. A., and Tyler-Smith, C. (1998). "A selective difference between human Y-chromosomal DNA haplotypes." Curr. Biol. 8(25): 1391-1394.

Kaplan, Robert D. (2005). "How We Would Fight China." Atlantic Monthly 295/5: 49-64.

Karban, Richard and Gregory English-Loeb (1997). "Tachinid Parasitoids Affect Host Plant Choice by Caterpillars to Increase Caterpillar Survival." Ecology 78: 603–611.

Kates, R.W. (1994). "Sustaining life on Earth." Sci. Amer. 271(4): 92-99..

Kilmer, Anne (1972). "The Mesopotamian Concept of Overpopulation and its Solution as Represented in the Mythology." Orientalia 41: 160-177.

Laherrère, J.H. (2000). "The Hubbert Curve: Its Strengths and Weaknesses." (Proposed to Oil and Gas Journal, Feb. 2000). http://dieoff.org/page191.htm.

Lambert, F. L. Fowler, RG, et al. (1988). "Will the Optimists Please Stand Up?". Population and Environment 10: 122-132.

Leach, D. (1981). "Re-Evaluation of the Logistic Curve for Human Populations." J. Roy. Stat. Soc. A144: 94-103.

Lederberg, J., Sharpe, R. E. and Oakes, P. C. (1992). Emerging Infections: Microbial Threats to Health in the United States. Washington DC: Nat. Acad. Press.

Livi-Bacci, M. (1989). A Concise History of World Population. Transl. by Carl Ipsen. Cambridge MA: Blackwell, 1992.

Loot, Géraldine, Stéphane Aulagnier, Sovan Lek, Frédéric Thomas, and Jean-François Guégan (2002). "Experimental demonstration of a behavioural modification in a cyprinid fish, Rutilus rutilus (L.), induced by a parasite, Ligula intestinalis (L.)." Can. J. Zool./Rev. Can. Zool. 80(4): 738-744

Lotka, A. J. (1925). Elements of Mathematical Biology. Williams and Wilkins, Baltimore; Dover reprint 1956.

Lutz, Wolfgang (ed.). (1996). The Future Population of the World: What Can We Assume Today? Revised and Updated Edition. London: Earthscan.

MacArthur, R. H. (1962). "Some Generalized Theorems of Natural Selection." Proc. Nat. Acad. Sci USA 48: 1893-1897.

MacIntyre, F. (1999). "'Is Humanity Suicidal?': Are There Clues From Rapa Nui?" Rapa Nui J. 13(#2): 35-41.

(Malthus B.c). denotes Book B, Chapter c of Malthus on Population, reproduced from the last (6th) edition of An Essay on the Principle of Population ... by Thomas R. Malthus (1798-1826), with an introduction by G. T. Bettany, London: Ward, Lock and Co., 1890.

McLaughlin, M. P. (1999). Regress+ A Tool for Mathematical Modeling. http://www.causascientia.org/software/Regress_plus.htm

Monro, J. M. (1993). “World population forecasts.” Nature 363: 215-216.

Mooney, J. (1928). Aboriginal Population of North America. Smithsonian Misc. Coll. 80 #7.

Moore, Jim (1999). “Population density, social pathology, and behavioral ecology.” Primates. 40: 5-26.

Moore, Janice (2002). Parasites and the Behavior of Animals. Oxford: Oxford University Press.

Moran, William J.(1971). “The Babylonian Story of the Flood.” Biblica 40: 51-61.

Pearl, R. (1924). Studies in Human Biology. Baltimore: Wilkins.

Pearl, R. and Reed, L. J. (1920). “On the rate of growth of the population of the United States since 1790 and its mathematical representation.” Proc. Nat. Acad. Sci. 6: 275-288.

Pearl, R. and Reed, L. J. (1930). “The Logistic Curve and the Census Count of 1930.” Science 72: 399-401.

Patzek, T. W. (2004). “Thermodynamics of the Corn-Ethanol Biofuel Cycle.” Critical Reviews in Plant Sciences 23(6): 519-567.

Pimentel, D., R. Harman, M. Pacenza, J. Pecarsky, and M. Pimentel (1994). “Natural resources and an optimum human population.” Population and Environment 15: 347-369.

Pimentel, D., O. Bailey, P. Kim, E. Mullaney, J. Calabrese, L. Walman, F. Nelson and X. Yao (1999). “Will Limits of the Earth's Resources Control Human Numbers?” Environment, Development, and Sustainability 1(1): 19-39.

Pohl, F. and Kornbluth, C.M. (1952). The Space Merchants. New York: St. Martin's Press.

Ratzel, F. (1897). Politische Geographie. (Munich: Oldenbourg).

Rekacewicz, P. (2001). “How the burden of the world’s refugees falls on the South.” Le Monde diplomatique, April 2001: 8–9.

Reynolds, V. and Tanner, R. E. S. (1983). The Biology of Religion. London: Longman.

Rummel, R. J. (1996). Death by Government. New Brunswick: Transaction Publishers.

Salter, Brian (2005). Mike Ruppert and Charles Galton Darwin. http://www.questionsquestions.net/docs04/ruppert_darwin.html

Schulze, P and Jack Mealy (2001). “Population growth, technology and tricky graphs.” May-June 2001 American Scientist pp 209-221.

Sen, A. (1981). Poverty and Famines: An Essay on Entitlement and Deprivation. Oxford: Oxford University Press.

Serrin, James (1975). “Is Doomsday on Target?”. Science 189: 4197.

Shawcross, Wm. (1986). Sideshow: Kissinger, Nixon, and the Destruction of Cambodia. London: Hogarth Press.

Simon, Julian (1981). The Ultimate Resource II: People, Materials, and Environment. Princetion University Press.

Smith, David A. and Lawrence C. Moore (2001). “World Population Growth”. Journal of Online Mathematics and its Applications, http://www.mathdl.org/mathDL/4/?pa=content&sa=viewDocument&nodeId=481&pf=1

Swift, Jonathon (1729). “A Modest Proposal for preventing the children of poor people in Ireland from being a burden to their parents or country, and for making them beneficial to the public.” In D. L. Purves The Works of Jonathon Swift, D.D., Dean of St Patrick’s, Dublin. Edinburgh: Nimmo, Hay and Mitchell, 1897: 475–478.

Taylor, Gordon Rattray (1970). The Doomsday Book. London: Book Club Association,,1972.

Tuckwell, H.C. and Koziol, J. A. (1992). “World population.” Nature 359: 200.

Umpleby, Stuart A. (1987). “World Population: Still Ahead of Schedule.” Science 237: 1555-1556.

Umpleby, Stuart A. (1990). “The scientific revolution in demography.” Population and Environment 11: 159-174.

[USBC] US Bureau of the Census (1970). Historical Statistics of the United States, Colonial times to 1970. Series Z-1. Washington DC: Gov’t Printing Office.

UN (2004). World Population Monitoring 2003: Population, education and development.

ST/ESA/SER.A/228

UNPD (2004). Population Division of the Department of Economic and Social Affairs of the United Nations Secretariat, World Population Prospects: The 2004 Revision

Verhulst, P. F. (1845). “La Loi d' Accroissement de la Population.” Nouv. Mem. Acad. Roy. Soc. Belle-lettr. Bruxelles 18: 1-38.

Von Foerster, H., Mora, P. M., and Amiot, L. W. (1961). “Population Density and Growth.” Science 133: 1932-1937.

White, Lynn, Jr (1967). “Historical roots of our ecological crisis.” Science 155: 1203-1207

Wrigley, E.A. and R.S. Schofield (1981). The Population History of England 1541-1871: A Reconstruction, Cambridge MA: Harvard University Press.

Zabel, Graham (2000). “Population and Energy.” http://dieoff.org/page199.htm.

Zimmer, Carl (2000). "Do Parasites Rule the World?" Discover, August.