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A Principal Component Simulation of Age-Specific Fertility – Impacts of Family and Social Policy on Reproductive Behavior in Germany

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Abstract

This contribution proposes a simulation approach for the indirect estimation of age-specific fertility rates (ASFRs) and the total fertility rate (TFR) for Germany via time series modeling of the principal components of the ASFRs. The model accounts for cross-correlation and autocorrelation among the ASFR time series. The effects of certain measures are quantified through the introduction of policy variables. Our approach is applicable to probabilistic sensitivity analyses investigating the potential outcome of political intervention. A slight increase in the TFR is probable until 2040. In the median scenario, the TFR will increase from 1.6 in 2016 to 1.63 in 2040 and will be between 1.34 and 1.93 with a probability of 75% under the most realistic policy scenario. Based on this result, it is unlikely that the fertility level will fall back to its extremely low levels of the mid-1990s. Four simple alternate scenarios are used to illustrate the estimated ceteris paribus effect of changes in our policy variables on the TFR as well as the results of simple extrapolations.

Keywords

Fertility, statistical demography, forecasting, family policy, principal component analysis, time series analysis

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1. Introduction

The future development of fertility is of great importance in many areas, particularly regarding planning for and evaluation of future needs for political intervention in family and social policy. The fertility level of a country is commonly represented by the total fertility rate (TFR), which is the sum of the age-specific fertility rates (ASFRs) over all ages during a specific year. Therefore, the TFR can be interpreted as the average number of children a woman bears during her reproductive phase given that the current ASFRs remain constant in the future. As this assumption usually does not hold for any woman, the TFR is considered to be a hypothetical measure (Bongaarts and Feeney 1998). For decades, the TFR in most parts of Europe has been well below replacement-level fertility (Eurostat Database 2018a), which is approximately 2.1 for these regions (Bujard 2015a; United Nations 2015; van de Kaa 1987).

Although births have no immediate effects, for example, on the financial balance of social insurance, they are the most important demographic factor in the long run. First, in a Bismarck-type social security system, the insured working population pays contributions from its labor income in every period. These contributions are then transferred to the recipients of social insurance payments during the same period (Graf von der Schulenburg and Lohse 2014:425–89). An example of this process, which is especially influenced by the aging of the population, is the pension system, where social security contributions are given to the retirees as pension payments. Another example is long-term care insurance, to which the working population regularly contributes, and is provided to the population in need of care. Low fertility in the long run leads to a ceteris paribus (c.p.) smaller workforce and a shift in the age structure in favor of older people. As a result, relatively fewer people of working age shoulder a higher financial burden in terms of social insurance for relatively more elderly people (Bujard 2015a; d'Addio and d'Ercole 2005). This effect is felt approximately 20 years after the birth year of a certain cohort, when it begins to enter the labor market. Small birth cohorts thus lead to shortages in the labor market (Bujard 2015b:136–139). Second, strong birth cohorts, such as the baby boomers in many European countries after World War II, in the long run lead to high demand for social insurance when they reach higher ages. Moreover, morbidity risks increase in old age, leading to higher health costs (World Health Organization 2015:95–98).

Forecasting future ASFRs therefore provides important quantitative information for political decision-making in response to these predicted trends (Zuchandke, Lohse and Graf von der Schulenburg 2014). For example, political decisions about pension system reforms must consider the future course of fertility in society and the resulting population structure. Family policy must attempt to address political measures to increase the TFR for low-fertility countries, which include the majority of Europe and East Asia (Bujard 2015a).

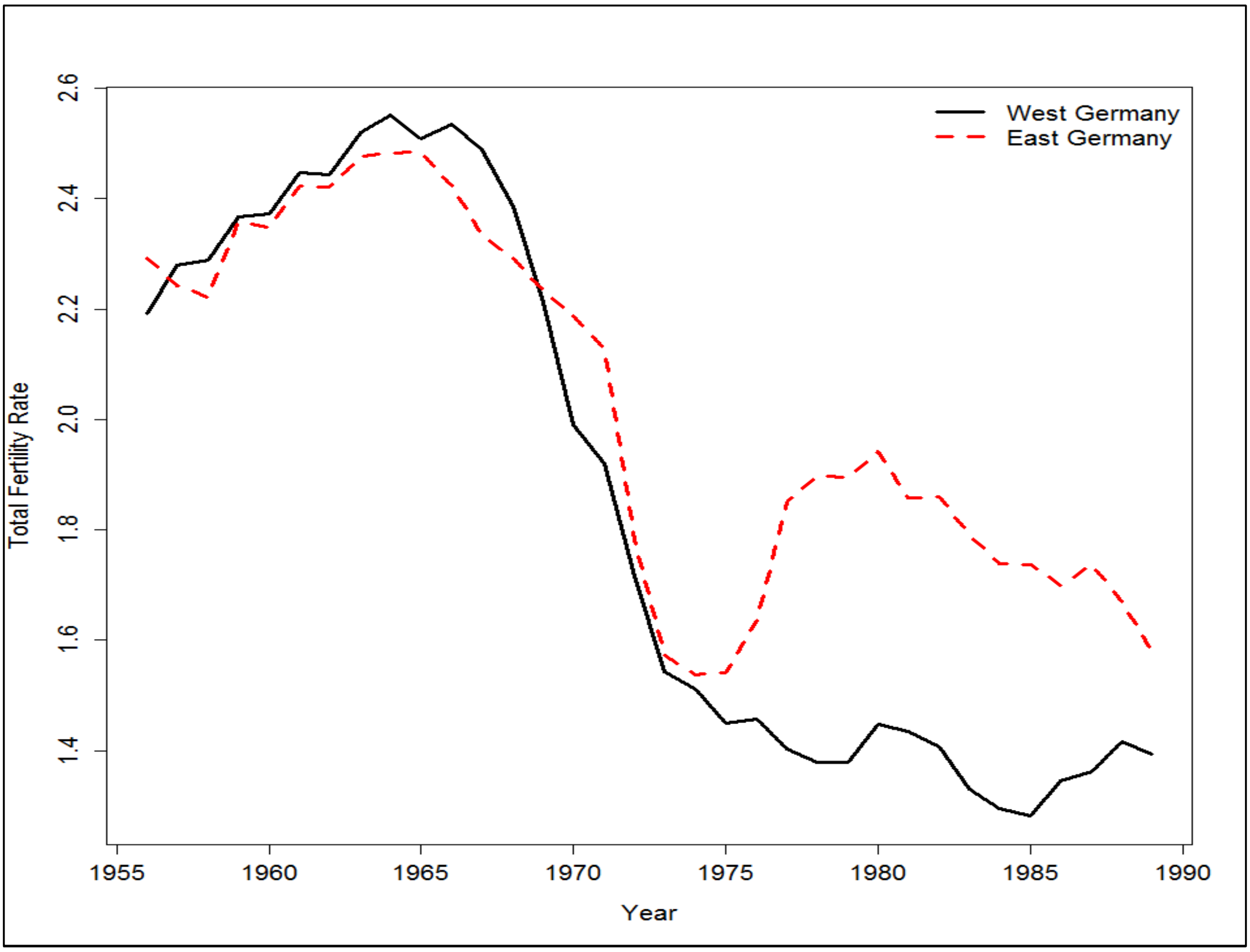
This contribution proposes a conditional forecast approach for the future course of ASFRs. The model framework is based on the Lee–Carter model for fertility (see Lee 1993), which makes use of principal component analysis (PCA) and time series (TS) analysis. We expand the framework by including family policy variables in the PCA, allowing a stochastic estimation of the potential future impact of political measures aimed at increasing a country's reproductive level. The methodology enables the integration of the correlations among the ASFRs and the autocorrelations among each set of ASFRs. Simulations of Wiener processes enable stochastic quantification of the future course of the ASFRs through prediction intervals (PIs). The trajectories of the ASFRs can be cumulated to stochastic forecasts of the TFR, which will be illustrated with 90% PIs.

In the next section, we give an overview of the fertility development in Germany since the mid-1960s, together with some sociological and juridical background. Section 3 provides a short review of past and present approaches for projecting or forecasting fertility in Germany, although many of these models are commonly applied internationally as well. Section 4 discusses the connection between family and social policy and a country's fertility level with a condensed literature review. In Section 5, we propose our forecast model. The model is in essence a two principal component time series model connected to the classical Lee-Carter model for fertility forecasting (see Lee 1992:321–323). The model indicates that fertility trends are strongly affected by the tempo effect in fertility (Bongaarts and Feeney 1998). Moreover, we conclude that the quantum of fertility can be influenced by adequate political measures. The model is applied for probabilistic sensitivity analyses of reforms in family policy based on a selection of variables available for Germany. However, the framework can be applied to other countries and should work even better for countries with longer TS.

2. Past fertility trends and reforms in family and social policy in Germany

In societies that follow a Bismarck-type principle in social security, low fertility rates over longer horizons are quite problematic. Small birth cohorts mean small cohorts entering the labor market when they reach the working age, which results in shortages in the labor market. Since that generation is obliged to pay the biggest part of the retirement income of the elderly population via contributions from their labor income, this leads to a high financial burden on the working population. This is especially difficult when small cohorts follow a phase of very high fertility, as has been the case for many European countries in the late 1960s and early 1970s. Figure 1 illustrates this situation by the sharp declines in the TFRs of East *(Deutsche Demokratische Republik or DDR)* and West Germany *(Bundesrepublik Deutschland or BRD)* after the mid-1960s.

Figure 1. Total fertility rates in West and East Germany



Sources: Human Fertility Database 2016a, 2016b; own design.

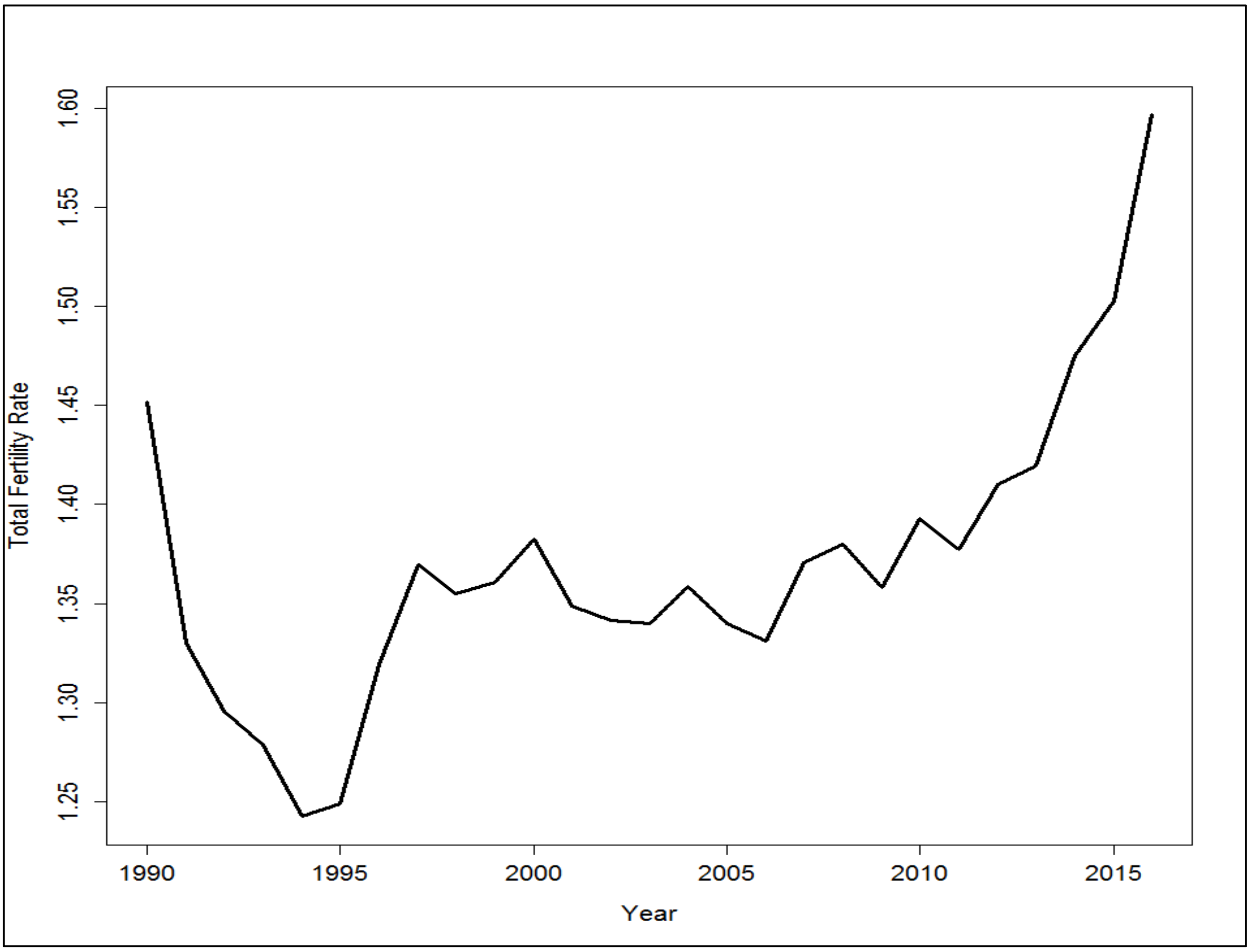
We observe increasing TFRs for both parts of Germany after World War II, reaching a climax in the mid-1960s, after which the TFRs decrease quite heavily until the early 1970s. The main reason for this was the second wave of the women's rights movement beginning in 1968, which was especially distinct in West Germany (Hertrampf 2008). The feminist movement has led to a postponement of births to an older age (Gustafsson 2001; Lesthaeghe 2010). Whereas this trend is persistent, it is important to determine the extent to which these postponed births are being recuperated by births in older age groups. Although the introduction of the birth control pill is not causally linked with the decline in births, the active decision to conceive children was facilitated by its market release at the beginning of the 1960s (Bundeszentrale für politische Bildung 2015). This trend persisted in the BRD until the late 1970s, and the TFR decreased to 1.38 in 1978 after reaching 2.54 in 1966. By contrast, the TFR in the DDR declined to 1.54 children by 1974, after which it increased to 1.94 in 1980.

Such strong fluctuations are problematic for countries following a Bismarck-style social security system, as explained above. Therefore, it is important to discuss options in family policy to influence the level of fertility. Whether these efforts indeed have an influence on fertility is subject to debate. This controversy exists in part because the effects of certain measures are difficult to specify since their impact can be observed directly only in the long run or not at all (Bujard 2011; d'Addio and d'Ercole 2005). For instance, statistical testing of the impact of policy in Germany is extremely difficult for several reasons. First, German reunification in 1990 caused a structural break. Second, time series (TS) of policy variables are not always available for long past horizons (e.g., due to reforms and the resulting structural breaks in the data).

There is a vast discussion in the literature about the effects of family policy on fertility. The conclusions differ strongly depending on the data source, the geographical units or countries under study, the methodology and the variables, as Bujard (2015a) has shown. For instance, the divergence of the trends in the two parts of Germany after the mid-1970s has been discussed in the literature. Temporal and infrastructural support in family policy in the DDR since 1972 were hypothesized to have an effect on the TFR. These effects nevertheless appeared to vanish after 1980 (Büttner and Lutz 1990; Höhn and Schubnell 1986). In 1979, the BRD, whose family policy had previously been restricted to financial support, began to alter its policy with the passing of the Maternity Leave Act (*Gesetz zur Einführung eines Mutterschaftsurlaubs)*. This law ensured maternity leave of up to six months after childbirth for employed women. Furthermore, the law forbade employers from releasing female employees during their maternity leave (BGBl I 1979/32). The next milestone in the undertaking of raising the TFR was the Federal Child-Raising Allowance Act *(Bundeserziehungsgeldgesetz or BErzGG*) in 1985. The BErzGG gave one parent, independent of sex, the opportunity to take paid parental leave of up to 10 months after the birth of a child (BGBl I 1985/58). This period was lengthened in the following years up to the first three years after birth (BGBl I 1989/32; BGBl I 1991/64). Figure 1 (previous page) shows an increase in TFR in 1980 and after 1985 – the years after the mentioned reforms – implying a positive effect of the family policy measures on reproductive behavior, although the effect might weaken after a certain period. Figure 2 (next page) shows the TFR for reunited Germany since 1990. After reaching its minimum in 1994 at approximately 1.24, the TFR has since recovered slowly.

In 2007, the BErzGG was annulled with the introduction of the *Bundeselterngeld- und Elternzeitgesetz (BEEG)*. Whereas the BErzGG offered a constant amount as financial compensation for the time spent raising a child, the parental allowance *(Elterngeld)* varies depending on the wage the person taking parental leave earned during the twelve months before the parental leave and can be up to two-thirds of the average wage of the person during that time span. The *Elterngeld* can be paid for up to 14 months

Figure 2. Total fertility rate in Germany



Sources: Destatis 2015a, 2015b, 2018c; GENESIS-Online 2018a, 2018b; own calculation and design.

for each couple, whereas the payment for one of the partners is limited to 12 months. An additional innovation was that the parental leave can be split among the first years of life of the child. Currently, the total allowable parental leave is up to three years taken over the first eight years after childbirth (BGBl I 2006/56). Thus, whereas classic measures in family policy have focused on financial compensation, measures since the late 1970s identified time as another important factor. The goal has shifted to giving employed potential parents the option to take time off work to raise a child without the risk of losing their jobs.

Following the *Trias of Family Policy* concept (Bujard 2011), family policy started to offer infrastructural support in childcare in addition to financial compensation and time for child raising. Since 1996, all parents in Germany have a claim for a childcare placement for their children over three years old (Spieß 2014). Whereas child care opportunities were rare in the past, especially for very small children under three years of age, in 2008, the *Kinderförderungsgesetz (KiföG)* was passed. Since August 2013, the *KiföG* has guaranteed childcare placement for children under three years of age if both parents are employed or in school (BGBl I 2008/57). Moreover, the government ran the *Zukunft Bildung und Betreuung* project (Future Education and Care), which, between 2003 and 2009, subsidized the construction of full-time schools and the evolution of normal schools to full-time schools (Bundesministerium für Bildung und Forschung 2018). These initiatives demonstrate the increasing awareness of the importance of giving both parents the flexibility to return to work relatively quickly after childbirth, and therefore, the high priority of coping with low fertility in family politics. The TFR has recently reached 1.60, its highest level since the early 1970s (for both parts of Germany combined). The question arises as to whether this trend stems from effective political measures.

3. Fertility forecasts and projections for Germany

Future projections for fertility in Germany are often based on deterministic scenario analyses. Germany's federal statistical office, *Destatis,* assumes a constant TFR in its 13th coordinated population projection for Germany. The underlying assumptions are that ASFRs will be decreasing for younger women under 30 years of age and that those losses will be balanced by increasing ASFRs for older women. This effect is known as the tempo effect (Bongaarts and Feeney 1998). These assumptions are based on survey data of the 2008 and 2012 micro-censuses on childlessness and the number of children mothers conceive in combination with historical trends of the final number of children for older cohorts. The trends between these two points in time represent the trends mentioned above. In an alternate scenario, which the authors classify as realistic, based on expert opinion rather than on empirical facts, they assume a slight increase in the TFR. This increase would result from a larger increase in the ASFRs of women over 30 years of age and constant fertility rates for younger women. In this case, we would not only see the tempo effect but also a quantum effect, resulting in an increasing TFR. The TFR in this scenario will increase to 1.6 in 2028 and remain constant thereafter[1] (Pötzsch and Rößger 2015). While the model assumptions might be based on reasonable arguments, they appear to be too restrictive for deriving long-term trends through 2060. Forecasting future trends into the far future is not advisable based on the very recent past. The longer the forecast horizon, the wider is the range of possible future scenarios since uncertainty arising in each period accumulates over time (see, e.g., Box, Jenkins, Reinsel and Ljung 2016:136–147). In simple terms: We know far more about the demographic behavior in the next year than about that in 30 years. Therefore, deterministic assumptions about future fertility become less and less likely the more distant into the future we are looking (Lee 1998:166–167). Furthermore, the large range of possible scenarios is neither identified nor quantified with individual probabilities.

The United Nations (UN) proposes a Bayesian hierarchical model (BHM) for stochastic projection of TFRs (United Nations 2015, 2017; Raftery, Alkema and Gerland 2014; Alkema et al. 2011). The BHM is a global model composed of 158 countries, which are classified into one of three possible cases: high-fertility countries (Phase I), countries transitioning from high to low fertility (Phase II), and low-fertility countries (Phase III). Germany is classified as a Phase III country because its TFR has been below the replacement level of 2.1 children since the early 1970s, as illustrated in Figure 1 and Figure 2. The TFR for Germany is assumed to slowly recover and converge toward 2.1 in the long run and is modeled by an autoregressive model of order one [AR(1)]. The quinquennial TFR is stochastically simulated 60,000 times with Markov chain Monte Carlo algorithms to identify the median scenarios with PIs through 2100. In the median scenario, the TFR in Germany is expected to exceed 1.6 through the mid-21st century and to exceed 1.7 by the end of the century. The trajectories for the TFR are thereafter distributed over the reproductive ages leading to trajectories of the ASFRs. These schedules are weighted averages of the past experience of low-fertility countries and Germany's latest historical development with respect to age-specific fertility. The fertility schedule is assumed to converge in the long term toward the global age-specific fertility schedule (Ševčíková et al. 2015). The UN model has some interesting features. It quantifies uncertainty via stochastic simulations while including both national and international trends. One might wonder whether the mathematical assumptions in the model are too restrictive in assuming a global convergence of international fertility trends. Similar points can be made about the ASFRs. The model proposed in our paper takes correlations among age groups into consideration without imposing excessively strict assumptions about their future behavior.

Alders, Keilman and Cruijsen (2007) attempt to combine the strengths of quantitative and qualitative models with TS models and perform TFR forecasting for 18 European countries, including Germany.

[1] Note that the TFR in Germany in 2016 has already reached this level, as explained in Section 2.

For Germany, they use data from 1950–2000 to estimate a generalized autoregressive conditional heteroscedasticity model resulting in point forecasts and 80% PIs from 3,000 trajectories until 2050. Given the estimated TFR, age schedules are used to estimate the ASFRs. The plausibility of the results of the quantitative forecasts are qualitatively assessed by two fertility experts. Although the technique appears to consider all necessities, the estimated 80% PI for the TFR in Germany is between 0.88 and 2.21, which is too wide for valuable policy implications. While we think it is better to overestimate future uncertainty instead of underestimating it, the 80% PI for the TFR appears to be overly conservative for Germany, where it has never been below 1.2 historically (Rahlf 2015) and has not surpassed a value of 2.2 since the early 1970s (see Figure 1). Another caveat is the assumption of an age schedule, which ignores the tempo effect.

Alho (1990) proposes indirect estimation of the TFR through forecasting the average ASFR. He constrains the average ASFR to upper and lower bounds through a modified logistic transformation. We will borrow that idea to some degree, as will be explained in Section 5. Bozik and Bell (1987) propose applying a PCA dimensionality reduction in estimating future ASFRs based on ARIMA forecasts of the TFR. Lee (1993) calculates a fertility index for indirect estimation of the ASFRs, which he derives from a PCA for the ASFRs. He integrates Alho's transformation into his forecast model to constrain the TFR to within certain bounds. To include uncertainty in the forecast, Lee applies a simple autoregressive moving average model [ARMA(1,1)], with which he simulates 1,000 trajectories for the fertility index. Approximate 95% PIs for the TFR can be derived with this process. Härdle and Myšičková (2009) apply Lee's model to forecast the TFR in Germany until 2060 with PIs. That method has some flaws. First, it assumes that the mean TFR remains constant at its last observed level, thereby ignoring the current fertility trends. Second, the PIs are rather narrow and have an unintuitive structure because they are wider for the first few periods and stagnate thereafter. Realistically, the risk in future predictions should increase for more distant points in the future, as explained above. Moreover, the restriction to one simple fertility index completely ignores the uncertainty associated with the remaining PCs, which leads to a systematic underestimation of future risks. Fuchs, Söhnlein, Weber and Weber (2018) use a similar approach to forecast the labor force in Germany until 2060. They distinguish between nationals and foreigners, thereby including possible effects of international migration on fertility. Hyndman and Ullah (2007) propose a robust adjustment to Lee's model that is insensitive to past outliers due to extreme events, e.g., wars or epidemics. Deschermeier (2015) applies this approach to forecast the population of Germany until the year 2030. A caveat of this approach is that outliers are assumed to be one-time events that cannot be repeated in the future. Because events that occurred in the past should not be ruled out for the future and are assumed to have zero probability, this approach leads to underestimation of future uncertainty in the forecast, which is discussed further in Section 5.

Since the 1980s, forecast models for cohort, rather than period, fertility have gained popularity. One such approach is the cohort autoregressive integrated moving average model of de Beer (1985). A modern approach based on Bayesian statistics is presented by Schmertmann, Zagheni, Goldstein and Myrskylä (2014). Although the cohort perspective has its advantages and is well justified, we prefer a period perspective because period effects can be observed in a more timely manner. Summary measures, such as the cohort fertility rate, provide clear information about the average number of children a cohort of females has given birth to. We are able to analyze these results only *after* the cohort's reproductive phase; therefore, there is a large time lag to consider.

4. Connections between fertility and family policy

Section 2 illustrated the complexity of family policy and the resulting difficulty of evaluating it for the case of Germany. Past studies have addressed these questions from theoretical or applied perspectives.

Economic factors have been found to effect the reproductive level of couples – at least since an investigation by Jaffe (1940) of United States (U.S.) Census Bureau data, which showed differences in fertility rates with U.S. census and tax data for the years 1800–1840. On a descriptive level, evidence has surfaced showing lower fertility in the wealthier strata of the population, estimated by the worth of personal properties the families owned (Jaffe 1940). Kiser and Whelpton (1953), in 1941, produced a more thorough investigation of the factors influencing fertility, entitled the *Indianapolis Study of Social and Psychological Factors Affecting Fertility*. The authors conducted surveys with 1,444 couples from which they tentatively tested 23 previously stated hypotheses on the connection of certain socio-economic, demographic, psychological and contraception factors on the planned family size. Becker (1960) built a microeconomic framework stemming from classical utility theory. Within this framework children are defined as a ‘good’ that provides utility to a household but also incurs costs (either direct monetary costs or indirect costs, such as time needed for child-raising). Assuming perfect control over the number of children a couple conceives, the partners will maximize their expected net utility by creating a portfolio composed of children and other goods. Walker (1995) discusses the effects of investment in social security systems on the TFR in a low-fertility setting, using the case of Sweden in the 1980s. For this, he proposes a life-cycle model in which, quite similar to Becker’s approach, the expected utility of females is maximized through optimal choice of timing of births and labor periods, which maximizes the present value of their lifetime utility, taking direct and indirect costs of children as well as statutory subsidies into account.

Given the presented theories alongside the extensive knowledge and accessibility of contraceptives, we would assume that couples on average would conceive the exact number of children they wish. Deviations from that number may be caused by biological infertility (Greil and McQuillan 2018:42–3) or infrastructural restrictions, such as excessive (opportunity) costs (Andersen, Drange, and Lappegård 2018; Testa and Bolano 2018) or the lack of childcare support (Andersen, Drange, and Lappegård 2018:900–2). Family policy could aim at facilitating these restrictions to help families reach their preferred number of children. Bujard (2015a) provides an overview of a range of international studies concerning the effects of family policy on fertility. Mostly, these studies attempt to estimate the impact via econometric models with the TFR as the endogenous variable.

Because a vast literature on this topic exists, we keep our focus on studies of fertility in Germany, especially in light of the lack of comparability for the impact of family policy across countries (see Kalwij 2010 on this). Cigno and Rosati (1996) investigated the influence of macroeconomic factors (income, male and female wage, interest rates) and family transfers in the form of child benefits and social security coverage on the TFR for Italy, the BRD, the USA and the UK using cointegration analysis. However, the analysis was largely incomplete because the model for the BRD did not include child benefits as an explanatory variable. One interesting result of the analysis was that social security coverage was negatively correlated with the TFR, indicating that a greater supply of social insurance has a negative effect on reproductive behavior. In a follow-up paper, Cigno, Casolaro and Rosati (2003) elaborated on the earlier model with a vector autoregressive model, using child benefit, social security coverage, pension gap, interest rate and mean real wage for both genders as explanatory variables. They identify positive correlations between the TFR and the lagged social security deficit. Moreover, the study supports the results of the earlier study; therefore, the generally good social security coverage in Germany leads to a reduction in the TFR, whereas a larger gap between working-age income and the received retirement pension supports the decision to procreate. In an investigation of micro-level data for 16 Western European countries, Kalwij (2010) applies a proportional hazard model to estimate the age-specific risk of child conception based on demographic, economic, and country-specific explanatory variables. He concludes that, among other effects, parental leave has a positive impact on the probability of bearing a first child, while subsidized daycare opportunities for employed mothers increase the likelihood to conceive an additional child if the mother had previously given birth. The results on financial benefits are ambiguous, which Kalwij traces to spurious regression when not controlling for the overall fertility, represented by the TFR and the crude birth rate, in the population under study.

Gauthier and Hatzius (1997) conduct regression of the TFR on economic and family policy variables for a pool of 22 industrialized countries and conclude that cash benefits have a positive effect on the TFR, whereas maternity leave opportunities have no effect on fertility. They attribute these results to the small variation in the maternity leave data. Adserà (2004) estimates a series of panel data models for data on 23 OECD countries for the years 1960-1997. She identifies, among other results, a highly significant positive effect of the length of maternity leave after birth on the TFR. Bujard (2011) performs a series of regression analyses for all OECD countries and finds that the financial benefits, the length of paternal leave and the rate of children under three years of age in daycare (lagged by one year) has a positive effect on the TFR. Furthermore, the analysis finds evidence that the costs of daycare are negatively correlated with the TFR, indicating that the marginal costs of children have a negative effect on birth rates (d'Addio and d'Ercole 2005). Bauernschuster, Hener and Rainer (2013) find, based on German panel data for the years 1998-2010, that a high degree of childcare coverage is associated with increased fertility rates for all age groups.

Using a micro-simulation approach based on panel data, Abiry et al. (2014:44–195) estimate the effect of different family policy measures on the birth numbers and cohort fertility alongside the female occupational behavior in Germany. For the *Kindergeld,* they quantify a positive, statistically significant effect on the short-term number of births as well as a long-term effect on the cohort fertility using a life-cycle model. Bujard and Passet (2013) estimate the effects of the *Elterngeld* reform in 2006 on fertility using SOEP data. They do not find a general statistically significant impact of the *Elterngeld* but conclude that it has a positive effect on the fertility of females over 35 years of age as well as for females with an academic degree. The reform therefore appears to have had an encouraging effect for recuperation of births in an older age group, especially for those with a high level of education, whose schooling takes a long time. Abiry et al. (2014:155–76) even derive a positive effect of that reform on fertility over all age groups with their micro-simulation. The authors, moreover, show that the insufficient supply of daycare spots and the costs these create for the parents lead to a significant decrease of the birth level in comparison to that envisioned by the potential parents.

The presented studies give an indication of the positive effects of different family policy measures on certain ASFRs and the TFR in general. These studies contribute to an understanding of reproductive behavior, but they cannot be operationalized in forecast studies that predict future reproductive behavior. The present study aims to fill this gap.

5. Method and data

The data used for this study are cumulated from multiple data sources to obtain a broad basis for our modeling approach. First, the live birth numbers by single years of age of the mother were obtained. Data for births since 1992 can be downloaded from Destatis' database, GENESIS-Online (GENESIS-Online 2018b). For the years 1968-1990, the age-specific birth numbers for the two parts of Germany were provided by Destatis on request (Destatis 2015b, 2018a, 2018b). The data for 1991 were also provided by Destatis for East Germany (Destatis 2018c), and the data for West Germany were taken from the Statistical Yearbook of the Federal Republic of Germany (Destatis 1993). The birth numbers for the years 1968 to 1991 were merged to avoid a structural break in the TS due to different geographical bases in the data. The population data by age and sex for Germany as a whole were provided by Destatis on request for the years 1967-2011 (Destatis 2016). The data for 2012-2016 were downloaded from GENESIS-Online (2018a). All the demographic statistics used were provided by Destatis, ensuring that no error in the data due to different sources exists. For the policy variables used in the next section, we extracted data on public expenditures for daycare from the *Kinder- und Jugendhilfestatistiken*, provided online by Destatis (Destatis 2004a, 2004b, 2005a, 2005b, 2005c, 2005d, 2005e, 2005f, 2005g, 2006a, 2006b, 2006c, 2006d, 2006e, 2009a, 2009b, 2009c, 2010; GENESIS-Online 2018e). Rates of inflation were calculated from the consumer price indices in Germany, which were downloaded from GENESIS-Online (2018c). The remaining policy variables were derived from the laws presented in Section 2.

Birth numbers in the data are broken down by single years of age and are cumulated for the upper age group of "50 years and older". To take very late births into account without overestimating the ASFRs for 50-year-old mothers, the ASFRs for females aged 50-54 years were extrapolated by cubic splines estimation under the conditions that the splines cut the ASFRs of the 49-year-olds, that the ASFRs of the 52-year-olds (as median age of the group) are equal to the overall ASFR for the age group 50-54, and that the ASFRs equal zero at age 55. Cubic splines interpolation (technically, we perform an interpolation between age 49 and age 55) over a certain interval fits a process assuming a cubic parametric function

$$f(a) = \beta + \gamma * a + \delta * a^2 + \varphi * a^3, \tag{1}$$

whose parameters (in Greek letters) need to be defined. A unique solution requires four conditions, of which two are given by the two knots (left-hand and right-hand sides of the curve). The third and fourth parameters are obtained by running an optimization algorithm. In our case, we apply the Hyman filter of the Forsythe-Malcolm-Moler algorithm.[2] The Hyman filter (Hyman 1983) ensures monotonicity of the spline, which is crucial for our approximation to avoid negative estimates for the ASFRs. Cubic splines are often found optimal for smoothing over age groups in demographic modeling since they give not only curves passing certain points but also smooth fits where adjacent splines have identical slopes and curvatures in the knots (McNeil, Trussell and Turner 1977:246–252; Deschermeier 2011:775–777). Live births to 13- and 14-year-olds were approximated in the cases where they were not available following an assumed geometric decrease:

$$\tilde{B}_{14-\alpha,y} = B_{15,y} * \gamma_y^{\alpha}, \alpha = 1,2 \tag{2}$$

Past live births $\tilde{B}$ for 13- and 14-year-old mothers in year *y* were estimated in this way. In this case, γ is the growth rate resulting from the following condition:

$$\sum_{\alpha=1}^{2} B_{15,y} * \gamma_y^{\alpha} = B_{14-,y}, \tag{3}$$

where $B_{14-,y}$ denotes the cumulated number of births to women aged 14 years or younger. The γ_y values were calculated mathematically under the condition that the estimators of the live births, which were rounded to zero decimal places, were equal to the known number of births in the age group.

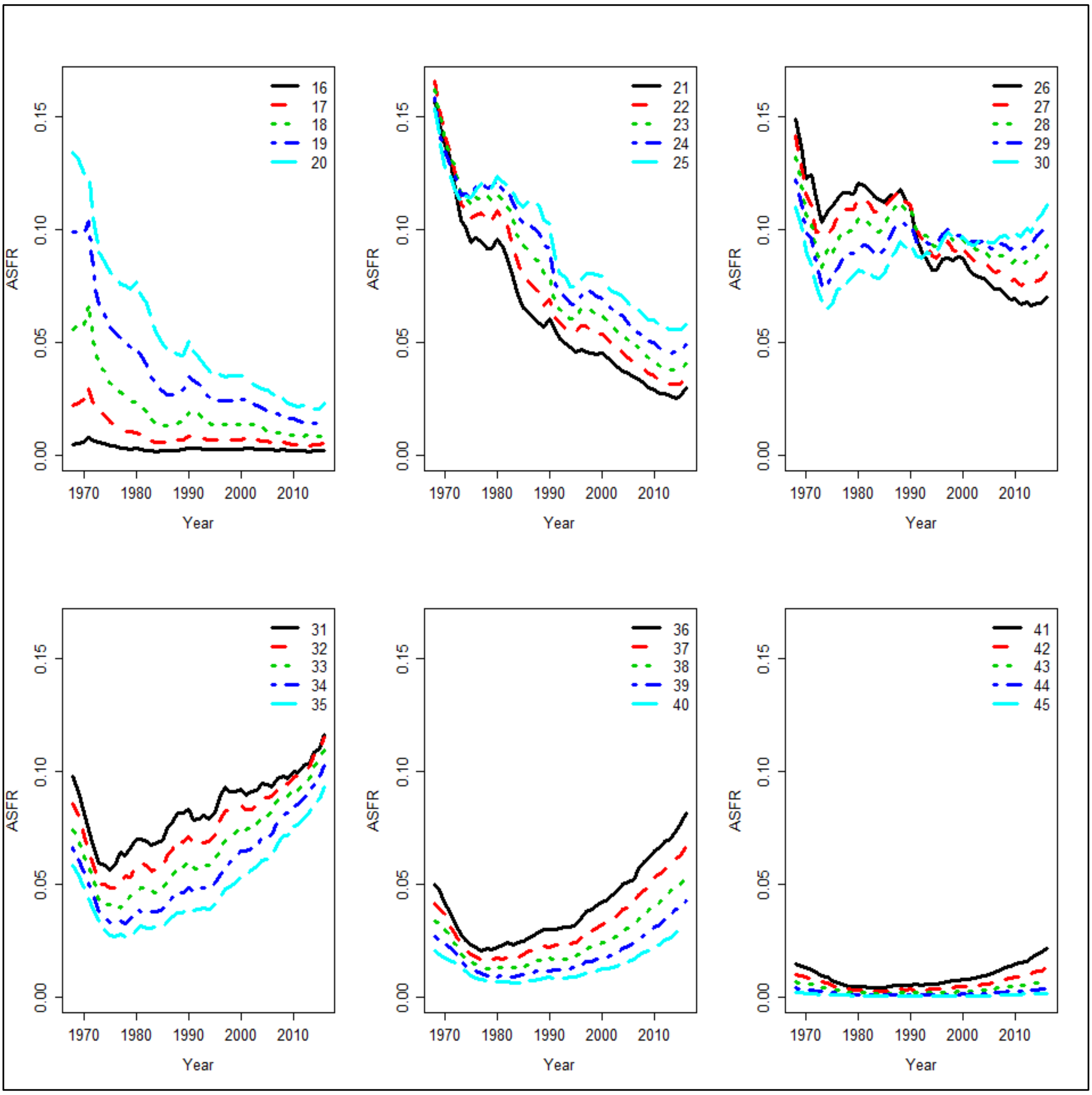
The ASFR for females aged *a* in year *y* was estimated as the ratio of live births to mothers aged *a* $(B_{a,y})$ over the mean female population aged *a* during year *y*. We have no daily, but only annual age-specific population data. To address this limitation, we assumed constant probabilities for death and migration over the course of the year, which allowed us to estimate the mean female population aged *a* in *y* $(\bar{F}_{a,y})$ as the mean between female population aged *a-1* at the end of year *y-1* $\left(F_{a-1,y-1}\right)$ and the female population aged *a* at the end of year *y* $\left(F_{a,y}\right)$:

$$ASFR_{a,y} = \frac{B_{a,y}}{\bar{F}_{a,y}} = \frac{B_{a,y}}{\frac{1}{2}(F_{a,y} + F_{a-1,y-1})} \tag{4}$$

[2] This is easily done using the **splinefun** command in **R**.

Figure 3 illustrates the ASFRs for 16- to 45-year-old females derived in this way.[3]

Figure 3. Age-specific fertility rates for females in Germany aged 16-45 years since 1968



Sources: Destatis 1993, 2015b, 2016, 2018a, 2018b, 2018c; GENESIS-Online 2018a, 2018b; own calculation and design.

We observe a general decline in fertility until the middle of the 1970s. Thereafter, the ASFRs for females in their mid-20s and older increased slightly, which might be associated with the reforms of 1972 and 1976 in the DDR (Büttner and Lutz 1990; Höhn and Schubnell 1986). The effect of these reforms appears to vanish after 1980. Since 1979, family policy in the BRD started to follow the concept of the *Trias* instead of providing pure financial compensation. Figure 3 shows that the positive trend in reproduction for females aged 30 years and older continued, but it remains unclear whether this is a result of the policy reforms since 1979.

[3] The ASFRs were derived for 13-54 year olds. For the sake of comparability, not all time series are shown here. ASFRs for very young and old females are too small for graphical illustration at the chosen scales.

PCA was performed as the next step. The use of the log or logit transformation of ASFRs and TFRs is popular in the literature (see, e.g., Alho 1990; Lee 1992) because these transformations ensure that future forecasts or projections remain within certain limits, as explained in Section 3. A standard logit transformation produces values in the interval (0,1) for the original variable.[4] We follow the logit approach for the ASFRs with upper and lower bounds according to Alho (1990:524). Since the historical ASFR maximum observed in our data (during the baby-boom period) was approximately 0.165, it appears reasonable to set the upper bound for the forecasts at $\frac{1}{6}$, meaning that annually, not more than every sixth female born in a certain period will have a live birth. Mathematically, this transformation is then

$$logit(r_{a,y}) = ln\left(\frac{r_{a,y}}{1/_6 - r_{a,y}}\right), \quad (5)$$

with $r_{a,y}$ being the ASFR of females in age a in year y. Algebraically, the PCA transformation is then

$$\boldsymbol{C} = logit(\boldsymbol{F}) \times \boldsymbol{E} \quad (6)$$

where $\boldsymbol{F}$ is a 49x42 matrix of the ASFRs (49 periods in the rows, 42 years of age in the columns), $\boldsymbol{E}$ is a 42x42 matrix of the loadings (each column is one eigenvector) and $\boldsymbol{C}$ is the theoretical TS of the principal components (a 49x42 matrix). Figure 4 (next page) shows the loadings of the first two PCs.

The first PC is loaded negatively for females under 29 years old, and the correlations become positive for females aged 30 years and older. Increases in PC 1 are therefore associated with decreasing ASFRs for younger females and increasing ASFRs for women over 30 years. Therefore, this PC is associated with the tempo effect. Increasing values for PC 1 show a strong trend toward shifting births from younger ages to older ages. Keeping in mind that a PC is simply a linear combination of the underlying variables (here: the ASFRs), we might also call it an index. Therefore, throughout the rest of this paper, PC 1 will be called the *Tempo Index*. PC 2 is generally non-negative. It takes negative values for the age group 25-27, but these are very small in absolute value (approximately -0.03). This PC therefore represents changes in overall fertility, which is why we refer to it as the *Quantum Index.*

Indices of the ASFRs enable the calculation of hypothetical past values of the PCs through plugging the past observations of the ASFRs annually into (6). To put the PCs into historical context, their hypothetical courses are illustrated in Figure 5 (p. 91).

Figure 5 shows an increase in the Tempo Index, which represents the persistent trend of birth postponement since the late 1960s. As described above, the postponement was a result of the second wave of the women's rights movement since 1968, manifesting as a female desire for emancipation and self-participation in the labor market. Gustafsson (2001) conducted a literature review and concluded from the analyzed studies and the TS of mean age at birth that the timing of births may have a large impact on the lifetime earnings and lifetime human capital of the mother. She concluded that the mother's childbearing tax was lower in cases of either a high or very low age of first conception in comparison to cases of intermediate ages of first conception. Therefore, giving birth at an age over 30 years is more attractive than giving birth at approximately 25 years if the mother chooses to pursue her own career.

The bottom picture in Figure 5 gives the course of the Quantum Index; the years 1979, 1985, 1992 and 2007 are marked. Major family policy reforms were introduced during these years (see Section 2). We observe increases in the index since 1985. After reunification in 1990, the Quantum Index decreased again until 1994, after which it mostly increased to the current level. Therefore, it appears to be associated with family policy reforms (see the overview of family policy reforms in Section 2).

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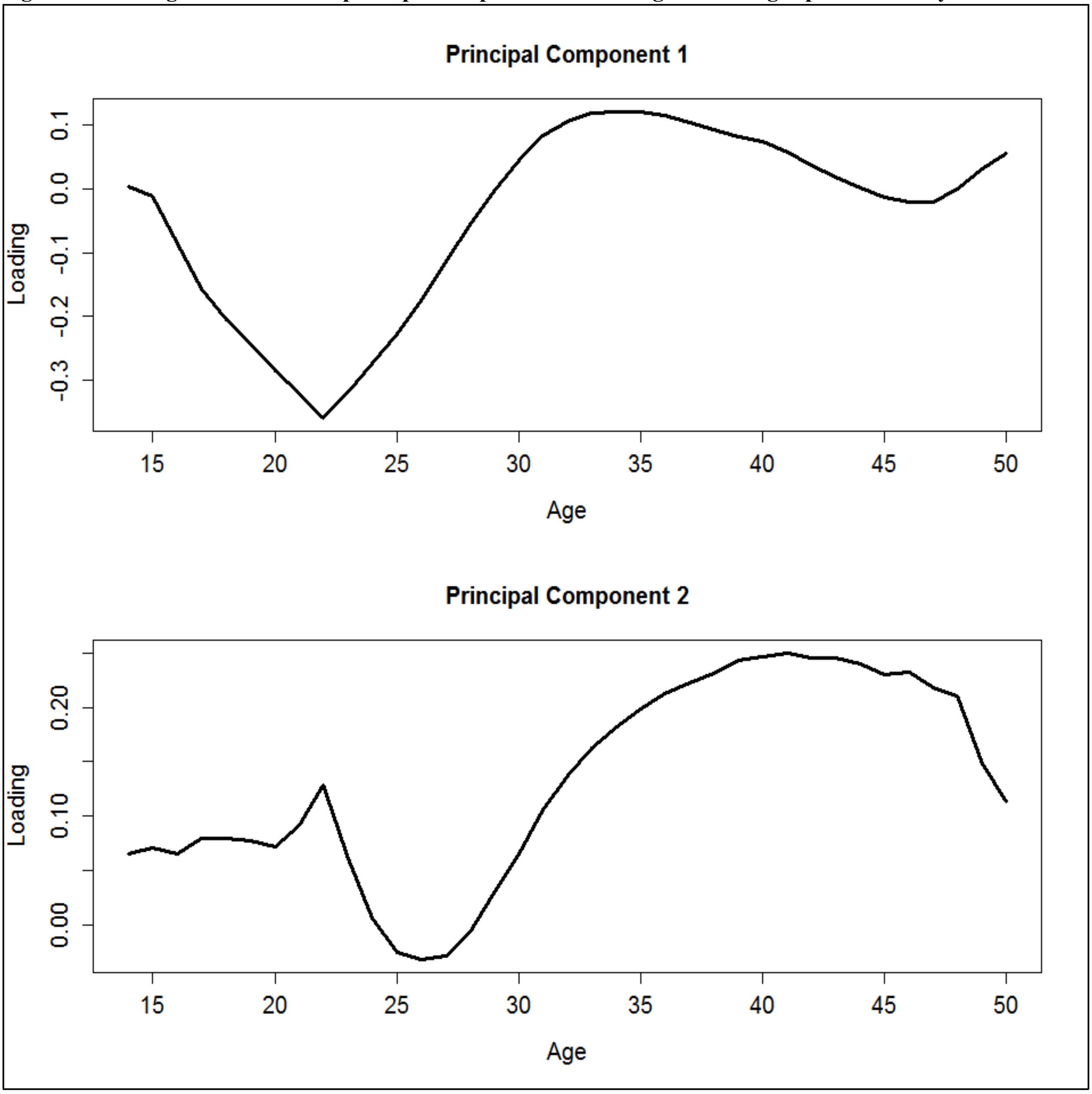
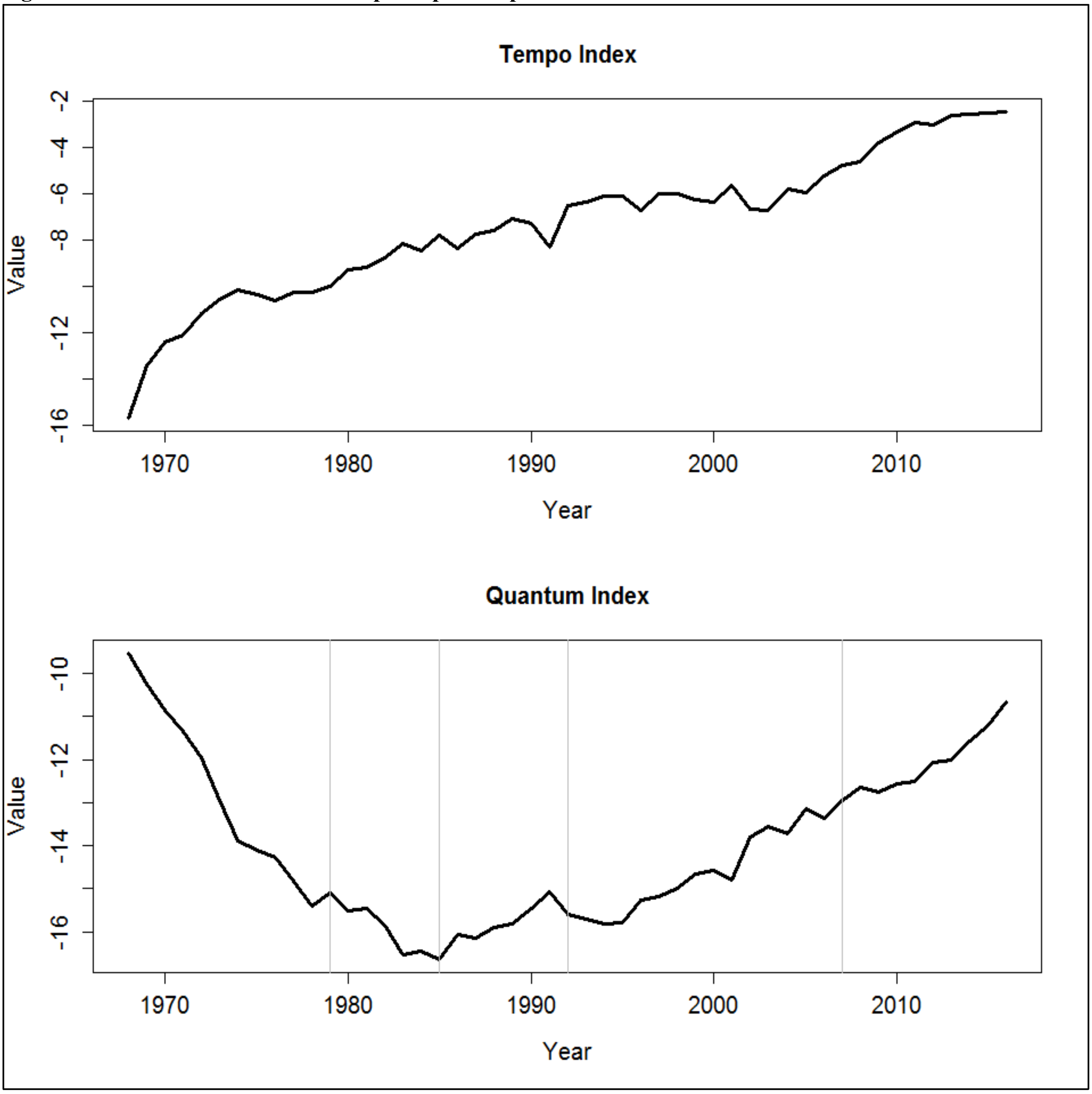
[4] The standard logit of a variable x is $ln\left(\frac{x}{1-x}\right)$; see, e.g., Johnson (1949).

Figure 4. Loadings of the first two principal components for the logits of the age-specific fertility rates



Source: Own calculation and design.

Figure 5. Past courses of the first two principal components



Source: Own calculation and design.

Three more scenarios will be considered; the reader can judge which scenario might fit reality best. For these scenarios, we perform a regression analysis with six possible explanatory variables to estimate the association of the Quantum Index to policy measures. The first category of explanatory variables includes financial benefits: the average annual child benefit *(Kindergeld – **KG**)* and financial benefits associated with parental leave with the old *Erziehungsgeld* ***(ErzG)***, which was discontinued in 2007. The amounts were calculated in Euros for the first year after birth, inflation-adjusted to 2010 prices. Its successor, the *Elterngeld,* was included as well. The *Elterngeld* is difficult to operationalize in a model because it depends on the previous wages of the persons receiving it, and it is also subject to upper and lower bounds. We attempted to measure its effect as a binary variable (***EG***) that takes the value of one for every year since the *Elterngeld*'s introduction and a value of zero otherwise. Finally, the pension entitlements for child raising were considered as indirect financial benefits (*Rentenpunkte – **RP***).We also tested an infrastructural variable. On the basis of the studies mentioned in Section 2, we assumed daycare to have an important impact on fertility. Whereas complete data on daycare are rare, TS on statutory expenditures for daycare coverage exist back to at least 1991. We used the real expenditures in billion Euros, adjusted to 2010 prices (*Betreuungsausgaben – **BA***). Finally, the impact of the length of parental leave *(Elternzeit – **EZ**)* in months as a time variable was included as well. The Quantum Index was regressed on the first lags of the explanatory variables because we can assume a time horizon of approximately one year between a couple's decision to conceive a child and its birth (see, e.g., Bujard 2011).

The quantified explanatory model could then be used for conditional forecasting of the Quantum Index under different policy assumptions. Three different c. p. scenarios were forecast. Scenario 3 (*Status Quo*) assumed constancy in the RPs and the EZ, which is very realistic.[5] All monetary variables were assumed to be held at their 2016 inflation-adjusted levels. This assumption is not totally unrealistic; moreover, this scenario gives us the opportunity to estimate the sensitivity of the final target variables, namely, the ASFRs and the TFR, toward investments in the respective variables. We addressed this question in the two remaining alternate scenarios, where we adjusted the future spending to estimate their effect on the Quantum Index and the TFR. Scenario 4 (*Child Benefit Adjustment)* assumed that the average *Kindergeld* would be adjusted annually with regard to inflation at the average rate since 1991. Scenario 5 *(Daycare Push)* presumed that the real investment in subsidization of daycare would be intensified at the same growth rate as has been the case since 1991. Both were c.p. scenarios assuming the effect on fertility to remain at the level quantified by the explanatory model, which might be doubted, as has been pointed out for the case of the Simple Extrapolation Scenario. The results will be presented in Section 6.

6. Results

Table 1 shows the results of the tested models. The estimated coefficients are presented for each variable along with their associated standard errors in brackets.

Table 1. Predictive model specifications for Quantum Index

Variable	Model 1	Model 2
(Intercept)	-21.474*** (0.5575)	-21.5131*** (0.5891)
Average Monthly Child Benefit [in €] (*Kindergeld – KG*)	0.0011*** (0.0002)	0.0009*** (0.0002)
Fixed Monthly Child-Raising Benefit [in €] *(Erziehungsgeld – ErzG)*	0.0005*** (0.0001)	0.0005*** (0.0001)
Income-based Monthly Child-Raising Compensation [in €] *(Elterngeld – EG)*	3.2602*** (0.6475)	3.1642*** (0.6824)
Total Pension Entitlement per Child [in Pension Points] *(Rentenpunkte – RP)*	- 0.34249* (0.19)	-
Annual Statutory Expenditures for Daycare Coverage [in billion €] (*Betreuungsausgaben – BA)*	0.16387*** (0.0254)	0.1551*** (0.0264)
Total Parental Leave Entitlement per Child [in Months] *(Elternzeit – EZ)*	0.051*** (0.0165)	0.042** (0.0166)

Source: Own calculation and design.

[5] Reforms of the parental leave are subject to political discussion, not regarding its total amount but, rather, how to split it.

A single asterisk next to an estimated coefficient means statistical significance at the 10% level, two asterisks are associated with statistical significance at the 5% level, and three asterisks denote significance at the 1% level.

In general, the regression results show strong connections between the policy variables and the Quantum Index. Model 1 shows a high impact of all regressors on the Quantum Index other than the pension entitlements, whose coefficient was significant only at 10% level, suggesting a lack of correlation. Moreover, a negative impact of this variable on fertility is theoretically implausible, since more RP for childbearing mean smaller marginal costs of the child, implying either a positive effect on the probability to conceive a child or no effect at all. A negative impact would economically make no sense. Omitting this variable in Model 2, all regressors show highly significant correlation to the Quantum Index with p-values of 0.02 (for **EZ)** and below. We therefore choose to take this model as a forecast model of that index. A large portion of the increase in the TFR observed during the last 20 years therefore appears to be attributed to effective family policy. Our estimates confirm the study results of Abiry et al. (2014) and Bujard and Passet (2013) on the effects of the *Kindergeld*, the *Elterngeld* and supply with statutory subsidized daycare. The positive effect of parental leave estimated here is in line with the international results reported by Kalwij (2010). As an OLS model of this type cannot completely explain the connection between the explanatory variables and the dependent variable, especially given the small data base and the lack of a control group usually given in this kind of research method, the errors in the fit are carried over into the nuisance of the forecast model, which is modeled using an ARIMA model, as will be explained below.

We simulate the ASFRs indirectly through conditional forecasting of the PCs. The Tempo Index has a stable positive long-term trend, which is statistically quantified by an OLS model with a logarithmic as well as a linear trend. The remaining disturbance after fitting that trend to the data is modeled with an ARIMA model[6]. The degrees of the ARIMA model are determined by graphical analysis of the hypothetical TS and its differences, the autocorrelation function, the partial autocorrelation function, the augmented Dickey–Fuller test and the ARCH-LM test for conditional heteroscedasticity.[7] The fitted model for the Tempo Index t is

$$t(y) = -14.7244 + 0.12135y + 1.4748 ln y + e_{y-1} + \varepsilon_y, \varepsilon_y \sim \mathcal{NID}(0, 0.51635^2), \quad (7)$$

where e is a random walk process[8] and y is the year. The uncertainty of the forecast is accounted for by simulating 10,000 paths of a Wiener process until the year 2040, following Vanella (2018). We can then derive quantiles from the simulation results. Figure 6 shows the median simulation as well as the estimated annual 90% PIs of the Tempo Index.

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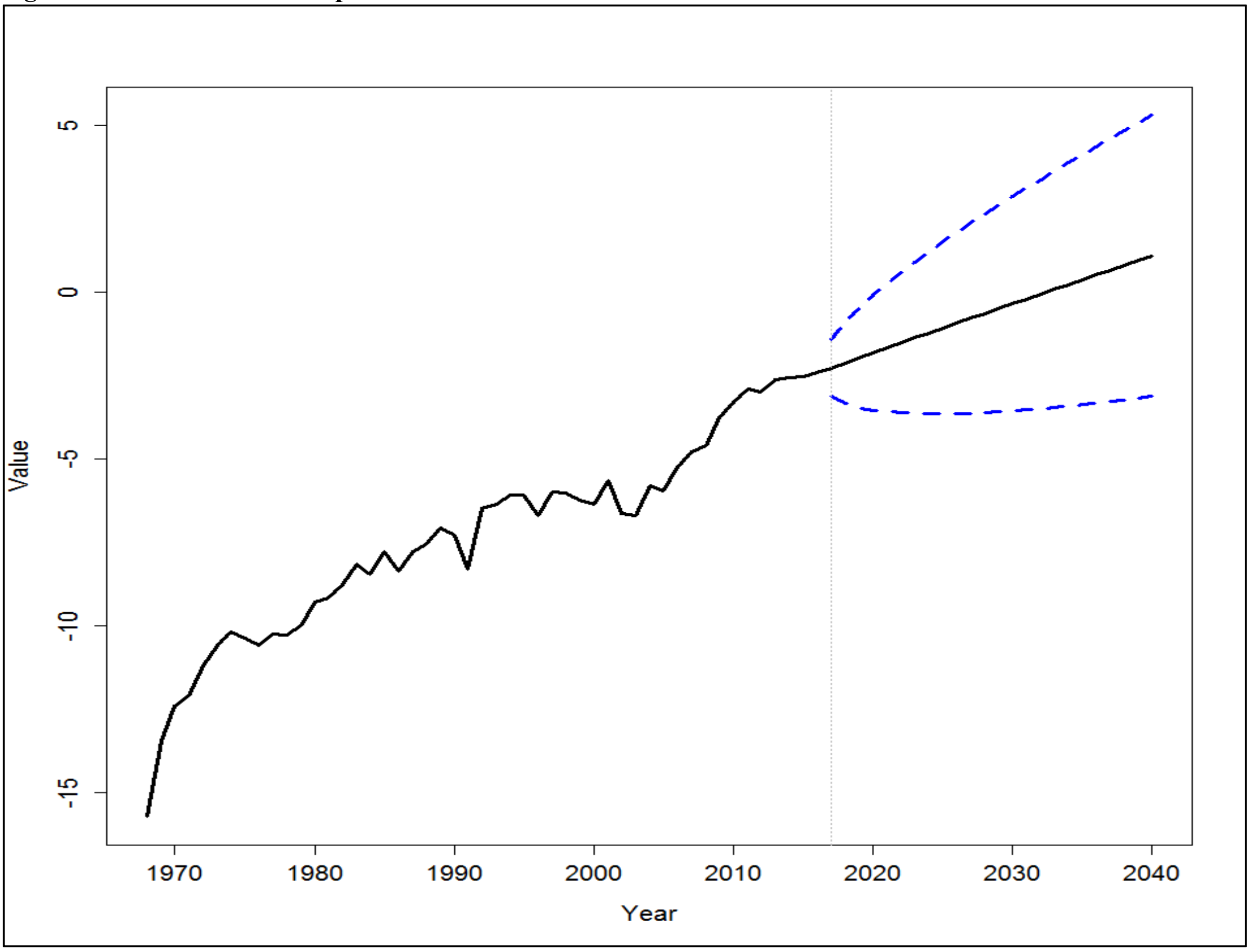
[6] See, e.g., Box, Jenkins, Reinsel and Ljung 2016:47–126; Shumway and Stoffer 2011:83–162; Vanella 2018:228–9 on ARIMA models.
[7] See, e.g., Vanella 2018:230–5 on these tests.
[8] A random walk is an ARIMA(0,1,0) process.

Figure 6. Forecast of the Tempo Index



Source: Own calculation and design.

We now turn to the conditional forecasts of the Quantum Index. For the Simple Extrapolation Scenario, the long-term trend and the stochasticity are fit using a similar procedure as for the Tempo Index. This assumes a progressive long-term trend, which is fit by a quadratic model; the resulting residuals by the tests mentioned above describe a random walk process:

$$q_1(y) = -0.4595y + 0.00315y^2 + f_{y-1} + \varphi_y, \varphi_y \sim \mathcal{NID}(0, 0.29194^2), \tag{8}$$

Regarding the Convergence Scenario, there are two questions to be answered before conducting the OLS regression. First, the inflection point of the logistic trend needs to be defined. Using graphical analysis of the Quantum Index and its second difference, which is approximately its curvature, no inflection point is found. The general trend of the curve is convex; therefore, we assume the inflection point to be the last observation in 2016. Second, the scale parameter α in Equation (9) needs to be estimated:

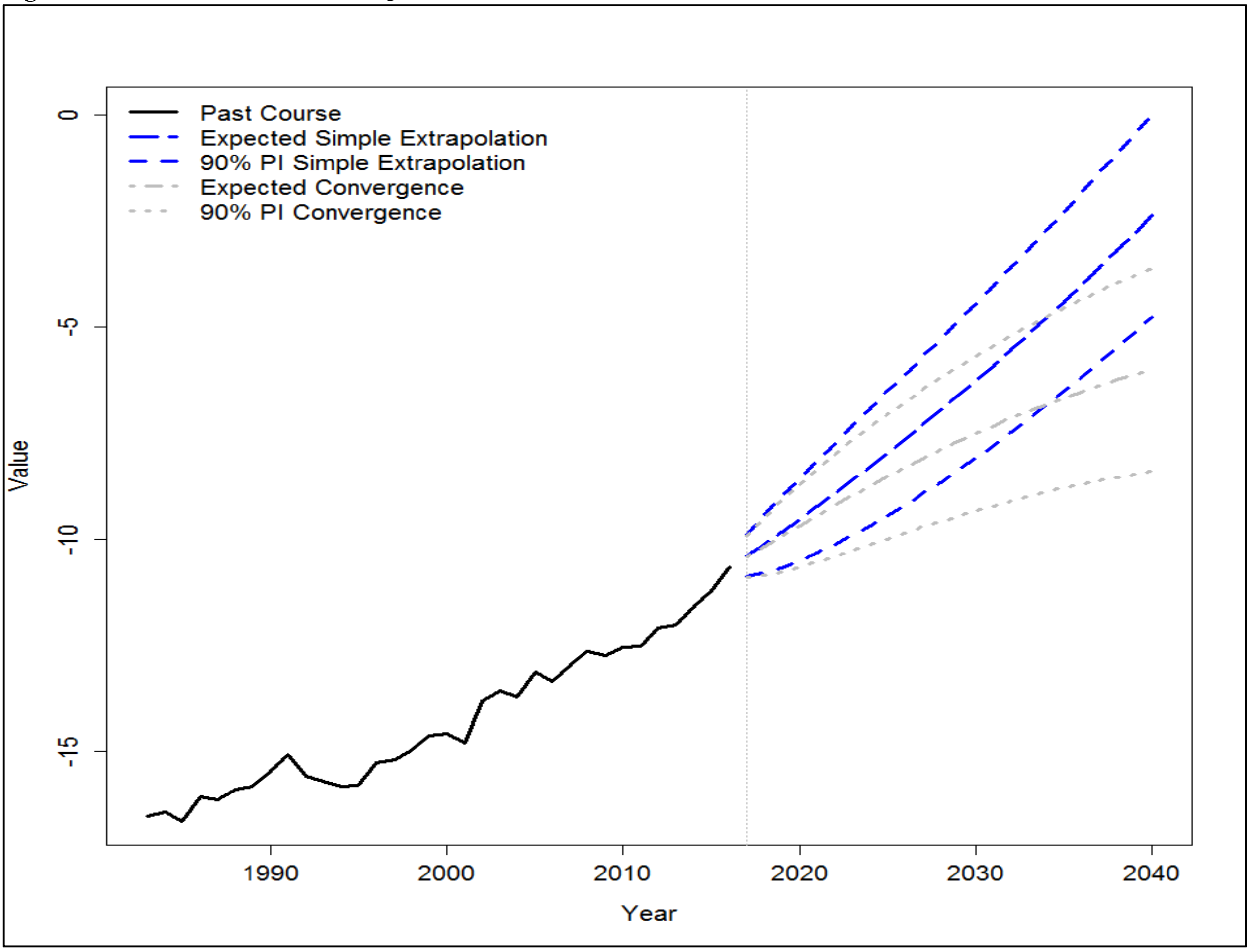
$$logit(x) = ln\left(\frac{x/\alpha}{1-x/\alpha}\right). \tag{9}$$

This can be achieved with maximum likelihood estimation to obtain the optimal scale with regard to the data. In this case, an α of 12.81756 leads to the best fit of the curve to the data. Using these two results, the OLS estimate for the forecast function is

$$q_2(y) = -17.34054 + logit^{-1}\left(\frac{y}{12.81756}\right) + g_{y-1} + \gamma_y, \gamma_y \sim \mathcal{NID}(0, 0.2927^2). \tag{10}$$

The slightly larger standard error in comparison to Scenario 1 represents the slightly worse fit of the model to the data. The conditional forecasts of the Quantum Index under these two scenarios with 90% PIs are given in Figure 7.

Figure 7. Conditional forecast of Quantum Index under scenarios 1 and 2



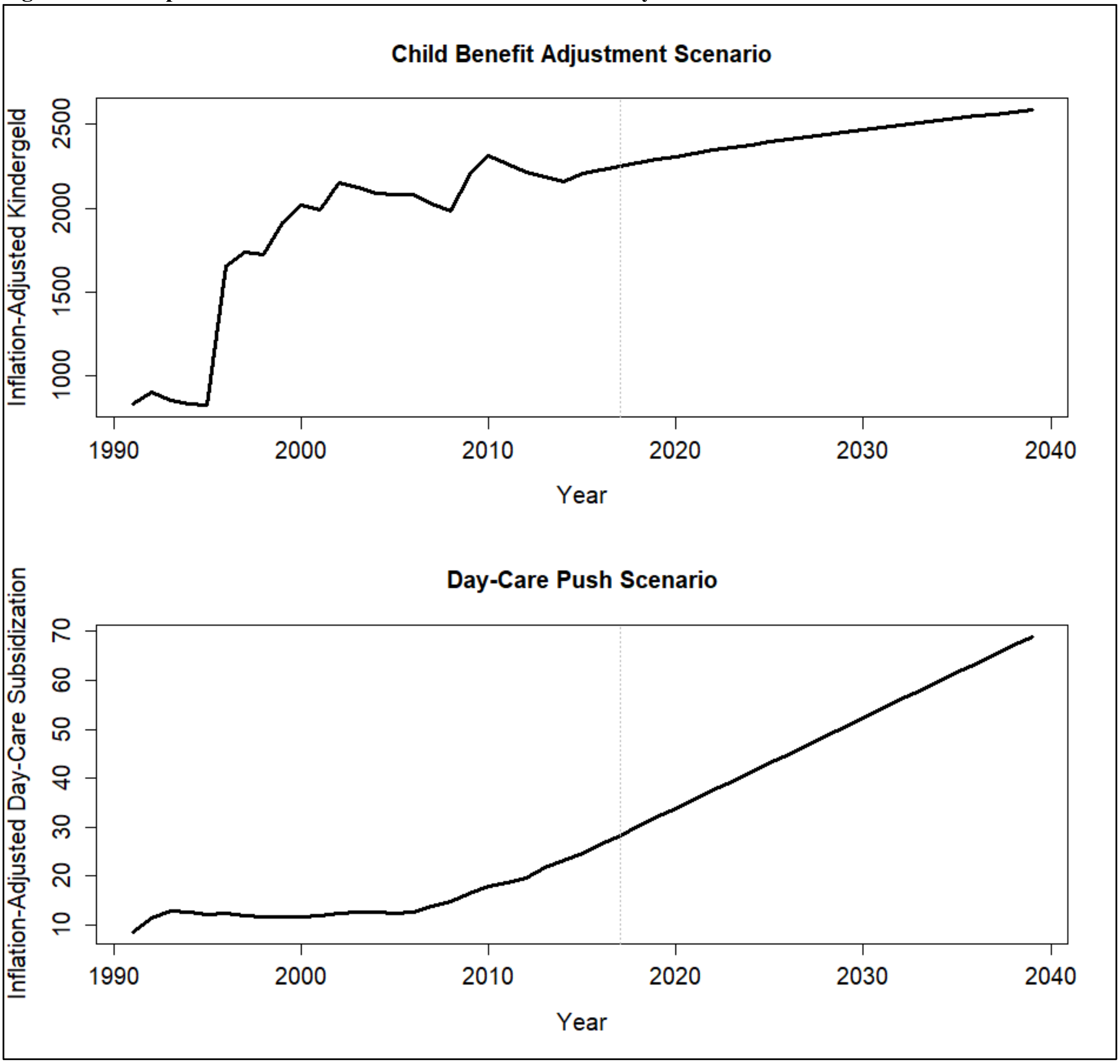
Source: Own calculation and design.

We now use the previously fit explanatory model to simulate the Quantum Index used for the remaining three scenarios. In this case, the nuisance parameter is again best fit by a random walk. Therefore, the forecast model of the Quantum Index is

$$q_y = -21.51312 + 0.00087kg + 0.00051erzg + 3.16415eg + 0.15511b + 0.04197ez + f_{y-1} + \xi_y, \quad \xi_y \sim \mathcal{NID}(0, 0.29747^2), \tag{11}$$

where f is a random walk process, kg is the annual average child benefit, $erzg$ is the annual *Erziehungsgeld* payment, and b is the money spent on subsidizing daycare in billion Euros. These monetary variables are all inflation adjusted to 2010 prices. eg is the *Elterngeld* dummy variable and ez the total entitlement to parental leave in months. All explanatory variables are lagged by one year. In the Status Quo Scenario, we estimate a conditional forecast assuming that the child benefit and daycare subsidization are held inflation-adjusted constant at their last observed levels in 2016. This assumption appears to be plausible and allows further sensitivity analyses. Scenarios 4 and 5 extrapolate the historical trends of the *Kindergeld* and the costs of child-care subsidization, respectively, inflation adjusted to prices in 2010. These inputs are predicted similarly as has been described above. These predictions are illustrated in Figure 8.

Figure 8. Assumptions about investment in child benefit and daycare for scenarios 4 and 5

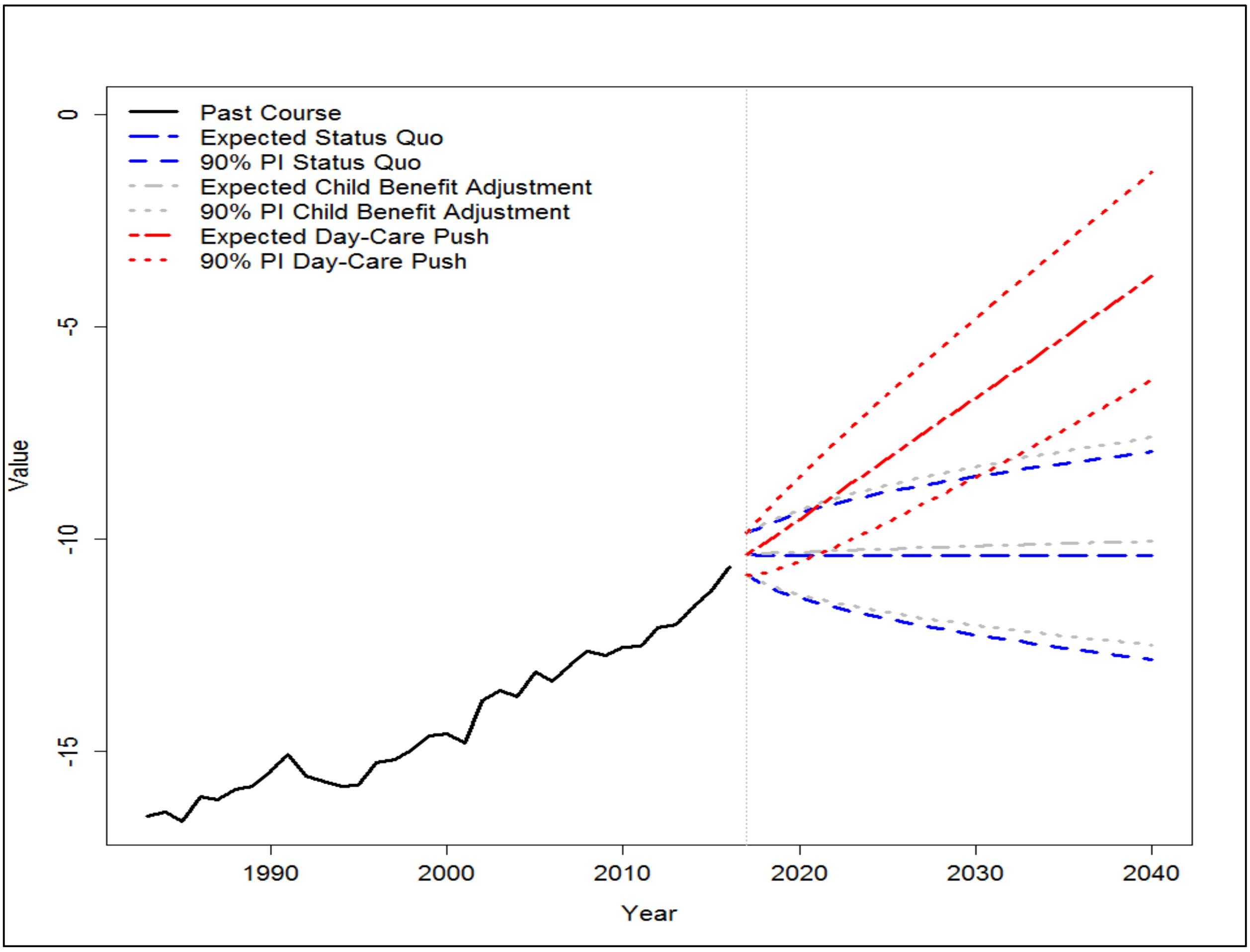


Sources: Destatis 2004a, 2004b, 2005a, 2005b, 2005c, 2005d, 2005e, 2005f, 2005g, 2006a, 2006b, 2006c, 2006d, 2006e, 2009a, 2009b, 2009c, 2010; GENESIS-Online 2018c, 2018d, 2018e; own calculation and design.

We should keep in mind that these predictions are simply examples to test the sensitivity of the Quantum Index toward adjustments in family policy. The policy measures are exogenous by nature, and an investment nearly three times as great as the current level into daycare is certainly questionable.

The conditional forecasts resulting from the simulation study for Scenarios 3-5, including the median outcome and the 90% PIs, are shown in Figure 9.

Figure 9. Conditional forecast of Quantum Index under scenarios 3-5



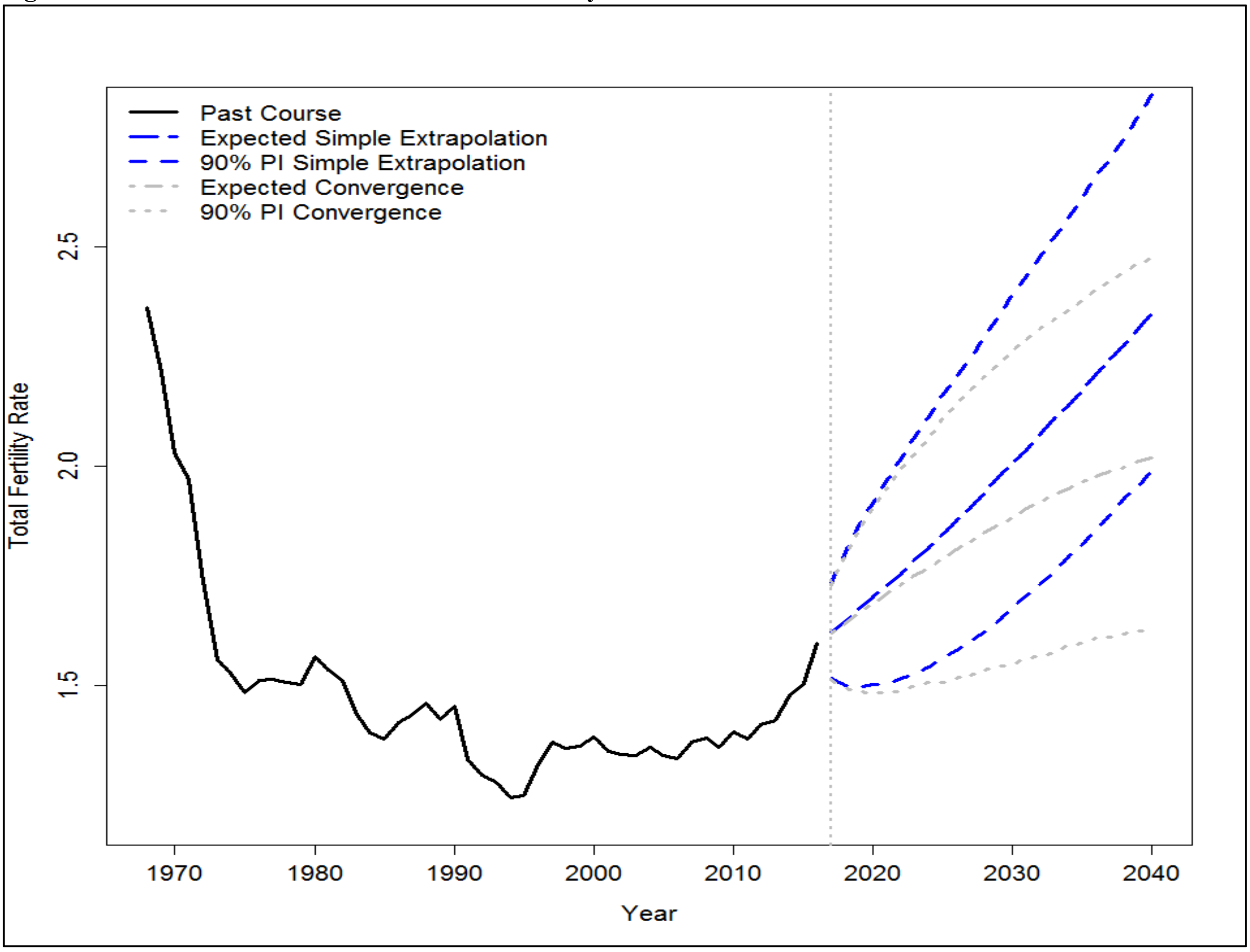
Source: Own calculation and design.

The first two PCs explain approximately 82% of the variance in the ASFRs for the base time horizon of 1968–2016. The Tempo Index explains the largest share (59.6%), whereas the Quantum Index explains 22.4% of the variance in the ASFRs. The remaining 40 PCs are simulated as random walk processes to consider their risk as well to some extent. This is an improvement on the classical Lee-Carter model, which would choose, e.g., the first two PCs and ignore the rest as irrelevant for the analysis. This is not problematic if the goal of the modeler is to predict the mean course, but if one wants to estimate the future uncertainty of the forecast as well, that model would in this case ignore about 18% of the total variance, leading to a systematic underestimation of future risk. Our model at least includes that uncertainty to some extent in the simulation study. A smaller number of PCs would lead to tighter PIs because a portion of the variance is simply assumed to be zero. Because the Tempo Index and the Quantum Index explain a large proportion of the variance in the data, the possible error arising from our random walk assumption is negligible. The resulting simulations of the PCs can be plugged back into (6) and solved for $\boldsymbol{F}$ to simulate the ASFRs. The simulation matrix of the ASFRs in year y is

$$\widehat{\boldsymbol{F}}_y = logit^{-1}(\widehat{\boldsymbol{C}}_y \times \widehat{\boldsymbol{E}}^{-1}) \quad (12)$$

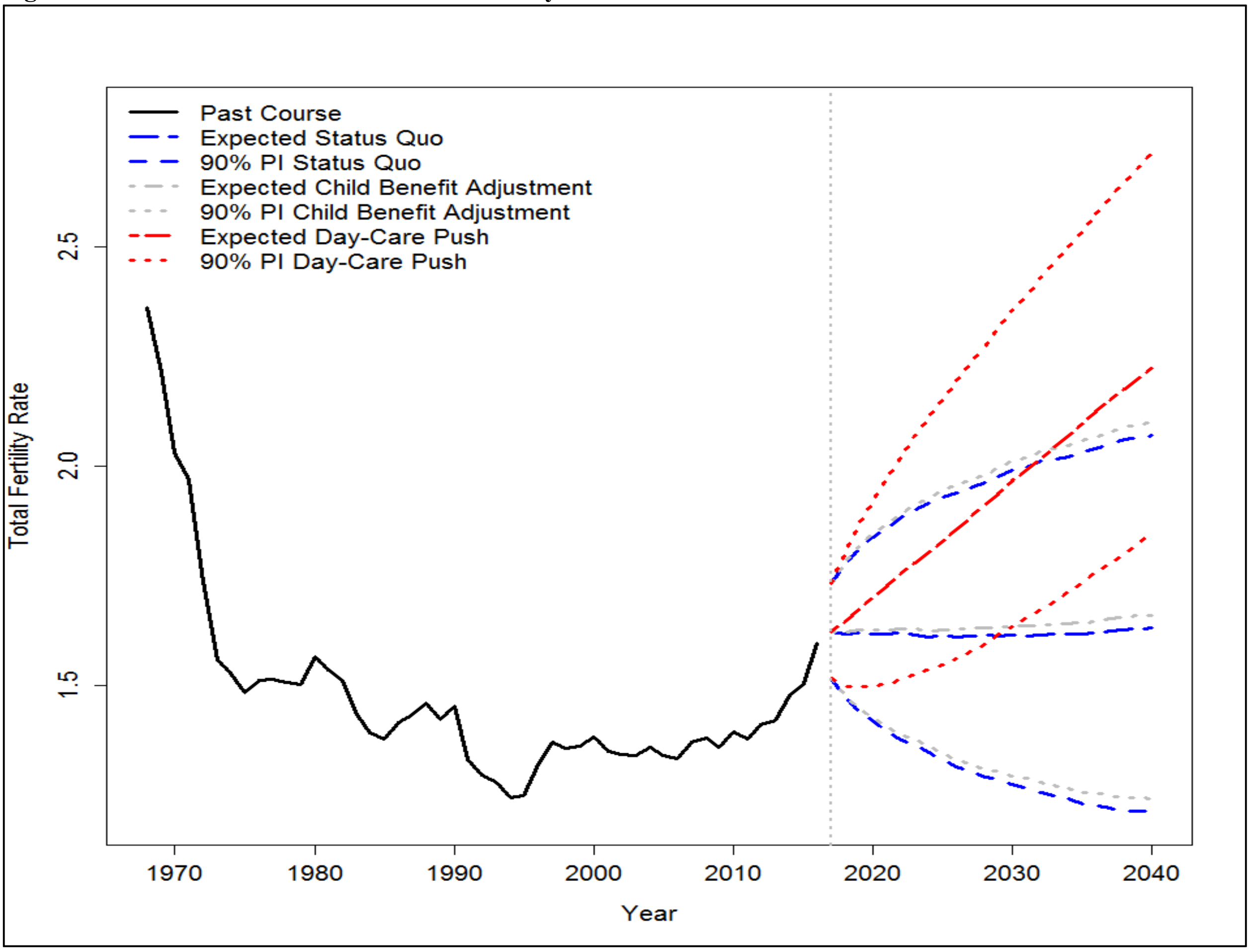
The hats over the matrix notations denote empirical matrices derived from simulations rather than theoretical matrices. The TFR is a synthetic measure that provides an idea about the general fertility level during a certain period. Therefore, we use our ASFR simulations to forecast the TFR, which is simply the sum of all the ASFRs for a specific year. Thus, summing over the rows of $\widehat{\boldsymbol{F}}_y$ yields 10,000 trajectories for the TFR in year y, from which we can obtain arbitrary quantiles. The conditional forecasts of the TFR with 90% PIs until 2040 under the five specified policy scenarios are illustrated in Figure 10 and Figure 11.

Figure 10. Conditional forecasts of the Total Fertility Rate with 90% PIs for scenarios 1 and 2



Sources: Destatis 1993, 2015b, 2016, 2018a, 2018b, 2018c; GENESIS-Online 2018a, 2018b; own calculation and design.

Figure 11. Conditional forecasts of Total Fertility Rate under scenarios 3-5

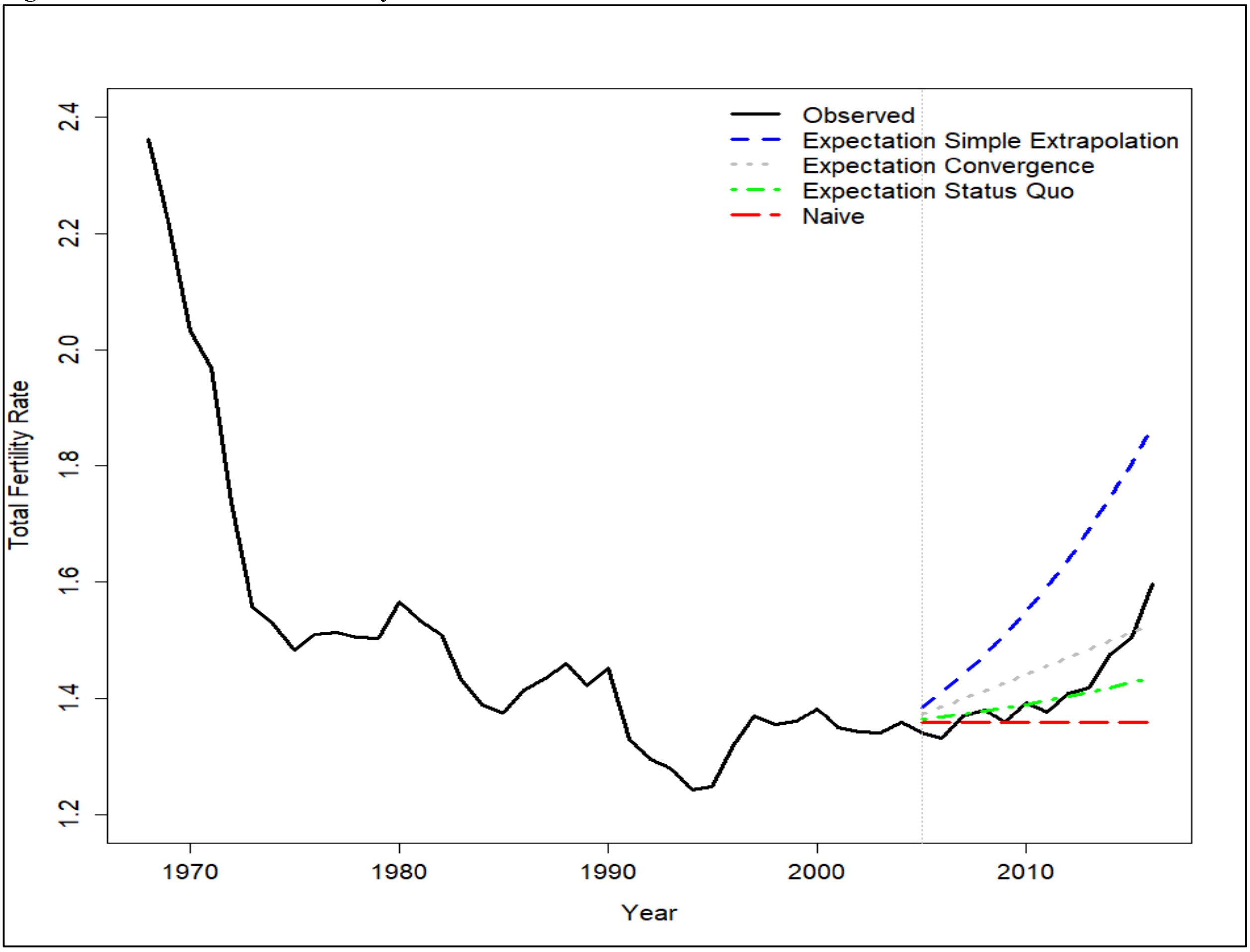


Sources: Destatis 1993, 2015b, 2016, 2018a, 2018b, 2018c; GENESIS-Online 2018a, 2018b; own calculation and design.

Whereas Scenarios 3 and 4 lead to a slight increase in the TFR until 2040 (1.63 or 1.66) from its level in 2016 (1.6), the remaining Scenarios 1, 2 and 5 show a huge increase to over 2 children per female (2.35, 2.02 and 2.22, respectively). Intuitively, Scenarios 3 and 4 appear more reasonable, given that the TFR in Germany has not exceeded a value of 2 for more than four decades. Moreover, the difference in the TFR between the level in 2016 and the all-time low of 1.24 in 1994 is approximately 0.36. Thus, a median increase of over 0.4 for the forecast to 2040 seems too high in the median scenario.

To tentatively assess the forecasting accuracy, we compare the predicted TFRs by backtesting following our Scenarios 1–3 and a naïve forecast to the observed TFRs for the years 2005–2016. In official fertility forecasting, naïve forecasts assuming a constant TFR are often preferred (see, e.g., Pötzsch and Rößger 2015:31–3). Therefore, we use a naïve forecast as the basis for comparison to our models. Scenarios 4 and 5 cannot be tested here because during the time interval used for the backtest (1991–2004), the EZ was payed and the EG was not yet introduced. Thus, the coefficients of our explanatory model for the questionable variables cannot be estimated. Figure 12 shows the historical course of the TFR with the predictions of the four mentioned approaches.

Figure 12. Predicted TFR for the years 2005–2016 due to different scenarios



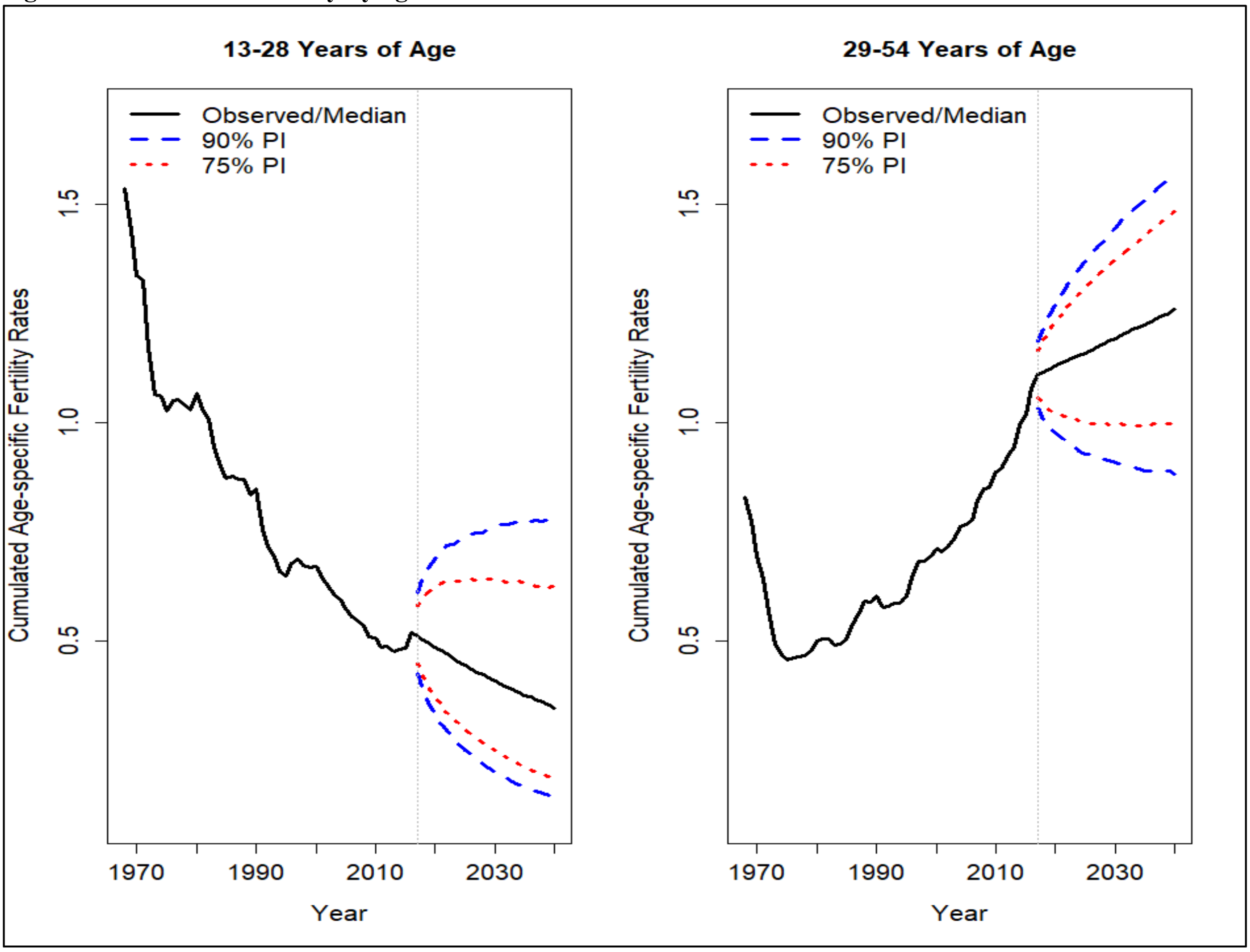
Sources: Destatis 1993, 2015b, 2016, 2018a, 2018b, 2018c; GENESIS-Online 2018a, 2018b; own calculation and design.

Graphically, the Convergence and the Status Quo Scenarios give the best fit to the data. Whereas the Status Quo Scenario overall predicts the course closest to the TFR observed in reality, the Convergence Scenario models the last years the best. A naïve forecast does not give a good fit, the Simple Extrapolation of the time series correctly predicts the direction, but completely overestimates the TFR, as assumed earlier. The standard errors of the four different models to the observed TFR are approximately 0.0914 for the naïve forecast, 0.1995 for the Simple Extrapolation Scenario, 0.0519 for the Convergence Scenario and 0.0556 for the Status Quo Scenario.

The reader can judge which scenario appears the most plausible for the fertility outlook. If our small test of accuracy gives some indication, the Convergence and the Status Quo Scenarios are the most accurate to forecast the future ASFRs in Germany. As noted above, a TFR of 2.02, as predicted under the Convergence Scenario, seems too high considering the historical course since the early 1970s and other recent projections of the TFR in Germany (e.g., Fuchs, Söhnlein, Weber and Weber 2018:45–6; Eurostat Database 2018c; United Nations 2017; Pötzsch and Rößger 2015:31–3; Vanella 2016:17). Therefore, we consider the Status Quo Scenario in greater detail. Under the assumed family policy regime, an additional slight increase in the TFR is probable over the forecast horizon. In the median scenario, the TFR increases from 1.6 in 2016 to 1.63 in 2040. Furthermore, the TFR will be between 1.21 and 2.06 with a probability of 90% and between 1.34 and 1.93 with a probability of 75%. Based on this result, it is unlikely that fertility will fall back to its extremely low level of the mid-1990s. An increase to the replacement level of 2.1 also appears to be unlikely, but an increase toward the Northern European level of approximately 1.8 (Eurostat Database 2018a) is realistic, though improbable.

Figure 3 (p. 88) showed that the long-term trends of birth rates are negative for women under 29 years of age and positive for women over 29 since the mid-1970s. These trends are expected to continue in the future, as shown by our forecast. Figure 13 presents the forecasts of the cumulated ASFRs for two age groups: 13-28 years of age and 29 years or older, underlying the Status Quo Scenario.

Figure 13. Forecast of fertility by age



Sources: Destatis 1993, 2015b, 2016, 2018a, 2018b, 2018c; GENESIS-Online 2018a, 2018b; own calculation and design.

We see that the postponement of births is likely recuperated at older ages, and the median result indicates an increase in the TFR. The slight increase in fertility for the younger group during the last two years is mainly associated with the high international immigration into Germany since 2014. In 2014 (Eurostat Database 2018b), 60,873 children in Germany were born to foreign mothers under 30 years of age out of the total of 236,413 mothers in that age group (a ratio of approximately 0.257). That same ratio increased to 71,146 out of 231,918 (0.307) in 2015 and even further to 92,581 out of 232,476 in 2016 (0.398). Because immigration is expected to decrease slowly in the future (see, e.g., Fuchs, Söhnlein, Weber and Weber 2018; Vanella and Deschermeier 2018), this increase in the ASFRs for younger females is expected to quickly revert to its long-term trend and the ASFRs will continue to decrease.

7. Conclusions, limitations and outlook

The future evolution of fertility is the strongest demographic driver for the long-term stability of social security systems and labor markets. Small birth cohorts ceteris paribus are associated with small cohorts entering the labor market, when they come of age, leading to shortages in labor supply. Moreover, in a Bismarck-type social security system a stable population structure is desirable since the birth cohorts after entering the labor market provide social security payments. These shortages in the labor market mean financial shortages in the social systems, which have to shoulder the burden of financing the older cohorts, which are then relatively strong since originating from stronger birth cohorts. Therefore, the importance of good fertility forecasts as a quantitative basis for political planning should not be underestimated. Possible political measures in family policy must be planned carefully, weighing the possible effects on fertility and the direct costs. Stochastic time series modeling combined with a simulation approach can help to visualize uncertainty via PIs. Official fertility projections usually implement a scenario technique, which does not provide information on the probability of occurrence. Political planning based on scenarios is limited to choosing the alternative that is most in line with the political agenda. The quantification of uncertainty makes the forecasting results of stochastic approaches less vulnerable to subjective treatment.

We proposed a principal component time series model for conditional forecasting of future ASFRs. The method is rooted in Lee–Carter modeling and accounts for autocorrelation and cross-correlation among the variables, thereby taking trends among the ASFRs and over time into account. This approach is a blend of the mentioned statistical methodology with commonly used qualitative scenario approaches, which take possible societal and political trends into consideration that are not covered by the past data completely. We have shown that a major portion of the fertility trends in recent decades can be attributed to the postponement of birth due to the second wave of the women's rights movement in combination with the development of the birth control pill. The postponement has also resulted in a recuperation of births in older age groups. Moreover, although there have been some effective reforms in Germany during the last 40 years, we conclude that good family policy measures have the power to compensate for social trends and can increase the probability of birth recuperation for women over 30 years of age. Our model also included family policy variables in the analysis and indirectly estimated their effect on fertility by age of the females via PCA. Our approach provides the opportunity for sensitivity analyses of possible family policy measures.

We want to stress that the analysis is not conclusive. The time series for the policy variables are relatively short, and the model fit will naturally improve as more data become available. Some variables, such as daycare supply, that would provide a more direct estimate of the effects of political measures on fertility are simply not available as regularly as needed. Other reforms, such as the BEEG in 2006, are difficult to operationalize because their structures are too complex to be covered completely in an econometric model. The model does not claim to completely cover all influences on the TFR; rather, it proposes an approach for further improvements in fertility forecasting in low-fertility countries by taking political intervention into account. Therefore, Scenarios 3 to 5 must be interpreted as conditional forecasts in the case that our identified explanatory model holds. This limitation is included to some degree in the model though since we consider the uncertainty of the forecast, represented by PIs. Other factors such as international migration can have an influence on fertility as well. These effects, which are not modeled explicitly by our model, are included implicitly in the simulations of the remaining 40 PCs, which were not explained and modeled in detail but are simulated as random walk processes. Thus, uncertainty arising from those factors is considered in our model as well to some degree. The implications of migration for fertility are difficult to assess in a time series framework due to the short time series available for policy variables and valid for migration in Germany and Europe. Thus, long-term trends of migration cannot be covered, at least for a data source such as Germany. Analyses for other countries or other regional units might consider that point as well if long enough time series without structural breaks are available.

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