Stochastic Modeling of Age at Menarche for Nepalese Girls and Developing Menarchial Life Table

Arjun Kumar Gaire, Yogendra Bahadur Gurung, Tara Prasad Bhusal

Population Review, Volume 63, Number 2, 2024, pp. 146-161 (Article)

Published by Sociological Demography Press
DOI: *https://doi.org/10.1353/prv.2024.a946252*

Population Review

Volume 63, Number 2, 2024

Type: Article pp. 146-161

Stochastic Modeling of Age at Menarche for Nepalese Girls and Developing Menarchial Life Table

Authors: **Arjun Kumar Gaire[1,2], Yogendra Bahadur Gurung[2] , Tara Prasad Bhusal[3]**

Corresponding author: Arjun Kumar Gaire < arjun.gaire@gmail.com>

Abstract

Menarche is one of the first and most significant milestones in a woman's fertility lifespan. It is a biological event in every girl's life; however, its timing is influenced by several biological and socioeconomic factors. Most studies on age at menarche have been mainly concerned with describing the statuses, differentials, and determinants. Skew distributions have been applied to fit the distributional pattern of age at menarche, and results are used to construct the menarchial life table. The objectives of this research are to explain the age at menarche of women by an efficient probability model, to identify the model that best fits age at menarche of women, to determine the characteristics of age at menarche of girls described by the model, cohorts analysis, and to construct a menarchial life-table to compute the expected age of menarche of girls. A secondary data set was taken to fit the skewed probability model, and different statistical tools were used to test and validate the model's fitting. The Skew-Log-Logistic model is found to be a significant fit for menarche data, and the results are applied to formulate the menarchial life table. The average age of menarche was estimated using the life-table technique for different cohorts. The distributional pattern of age at menarche is changing from a normal to skewed curve, and the mean age at menarche has decreased for recent cohorts. The Skew Log-Logistic model is a significant fit for Nepalese females' distributional pattern of menarche data, and the results were applied to formulating the life table.

Keywords: Log-logistic, Menarchial life table, Nepal, Skew

[1] Department of Science and Humanities, Khwopa Engineering College, Purbanchal University, Bhaktapur, Nepal
[2] Central Department of Population Studies, Tribhuvan University, Kirtipur, Nepal
[3] Central Department of Economics, Tribhuvan University, Kirtipur, Nepal

Introduction

The onset of the first menstrual cycle, known as menarche, and the natural cessation of menstruation, referred to as menopause, are key stages in a woman's reproductive life. These two events are considered milestones in the reproductive life span of women. Kisch (1910) was the first to use menarche in medical science. For the assessment of the developmental status of pubertal girls, menarche is one of the maturity indicators (Cameron and Nagdee 1996). Menarche is the most important biological event in the life of every girl. However, its timing is influenced by several biological and socioeconomic factors. Menarche signals the beginning of fertility and reproductive capabilities, so the absence of it should prompt the healthcare professional to perform a pathology evaluation Lacroix et al. (2022). The age at menarche is a crucial step in the reproductive life span of women. The historical record of Europe shows that the mean age at menarche (MAM) declined from about 16.5 to about 12.5 years over the past two centuries. The urban population shows an earlier decline in MAM than the rural (Lehmann et al., 2010).

Furthermore, few research studies describe the timing of age at menarche. Hence, the main aim of this research is to propose a flexible probability model to describe the age at menarche for Nepalese girls. Observing the right skewed nature curve of Nepalese girls' menarche, skewed probability models were used to fit the distributional pattern. The fitted results were applied to construct the Menarchial life tables.

Most studies on the age at menarche of girls have focused on determining statuses, differentials, and determinants. Socioeconomic status, genetics, general health, nutritional status, exercise, seasonality, and family size are frequently discovered determinants of girls' menarche; however, they are constantly being researched. Limited studies have been confined to finding the distributional pattern using a probability distribution. Some research focuses on the mean age of menarche and the relationship between age at menarche and biological and socioeconomic factors. The mean age of menarche for Indian rural girls was found to be 12.9 years (Khan and Hallad, 1994).

Similarly, Raji et al. (2006) found that the mean age at menarche for southwestern Nigeria among the 542 randomly selected female university students was 13.66 years. Furthermore, Akther and Islam (2012) found that the mean age at menarche for school-going Bangladeshi girls of classes five to eight was 12.32 years. They discovered that taller, higher body mass index (BMI) and heavier girls reached menarche earlier than shorter, thinner, and lower BMI. Furthermore, they found that height, weight, father's income, BMI, time spent watching television, and the father's education were all associated with early menarche in school-aged girls. Garenne (2020a) states that the median age for menarche in African countries ranges from 13.1 to 15.2 years. The variation was associated with countries' development indicators and focused on the two variables of food intake and urbanization. Similarly, the MAM for sub-Saharan African countries was 14.5 years, and for North Africa, it was 13.5 years, with an average of 0.47 years of reduction of MAM over the 30-year periods (Garenne, 2020b).

A similar and consistent result was found in Nepal, as Aryal (2005) estimates the mean age of menarche as 14.6, 14.3, and 13.9 years for an early, intermediate, and recent birth cohort in the Palpa and Rupandehi districts of western Nepal. Likewise, Ghimire and Ghimire (2014) estimate the mean age of menarche as 12.4 years in the Palpa district of Nepal and relate the influencing factors to the socioeconomic status of girls, their age at first sexual feelings, and occupation. Aryal (2011) establishes the statistical association between the age of menarche and other fertility events in women's lives, such as marriage, the first birth, and menopause. From this evidence, the mean age of menarche in Nepal is consistent with global figures and sometimes slightly higher. Menarche is regarded as early if it happens before or at the age of ten and late if it happens at or after fifteen (De Sanctis et al., 2019).

Researchers in Asian countries have paid little attention to the mathematical modeling of demographic variables; however, modeling these variables has received great attention among demographers worldwide (Gaire and Aryal, 2015). Fertility, one of the key demographic indicators other than migration and mortality, is directly affected by the age of menarche and menopause. Researchers used different models to describe age timing at menarche: Mascie-Taylor and Boldsen (1986) observed age distribution at menarche as a deviation from normality for British girls. They used a mixture of three normal distributions with equal variances but different means. Aryal (2005) applied the logistic probability distribution to describe the distributional pattern of age at menarche by using retrospective reporting data from rural Nepal. In this research, by observing the right skew nature of the distribution of age at menarche, we proposed alternative models, log-logistic (LLog) and skew log-logistic (SLLog) distributions, to describe the distributional pattern of Nepalese girls according to their age at menarche. The SLLog model is found to fit the Nepalese data statistically better than other models. The objectives of this research are to find a suitable probability model to fit the distributional pattern of age at menarche of Nepalese girls, cohort analysis of different birth cohorts of girls, and apply the results to construct the Menarchial life tables for different cohorts and estimate the expected age of menarche.

Methods and Data

The secondary data for this study was taken from a cross-sectional survey entitled "Demographic Survey on Fertility and Mobility" conducted by Aryal (2005) in the Palpa and Rupandehi districts of Nepal. It consists of 811 households comprised of a sample of 1019 married females of marriageable age and 547 unmarried girls. Three probability distribution models have been used to fit the distributional pattern of the age of menarche for the Nepalese girls for three different birth cohorts: early (before 1970), intermediate (1970-1979), and later (1980 and after), and the results are presented in Table 1.

Table 1: Observed number of girls with age at menarche for different cohort

Age of Girls	Early (Before 1970)	Intermediate (1970 – 1979)	Later (1980 and later)	Total girls
12 – 13	34	39	45	118
13 – 14	78	90	117	285
14 – 15	110	122	125	357
15 – 16	94	92	69	255
16 – 17	60	49	31	140
17 – 18	29	22	10	61
18 – 19	15	10	2	27
19 – 20	5	2	2	9
20+	4	2	-	6

In this article, we use quantitative data on the age of menarche in girls. We try to establish the probabilistic relationship between the age of girls at the time of menarche as the independent variable and the occurrence of menarche as the dependent variable. The quantitative data of age at menarche were used to analyze and fit the distributional pattern, so the ontological position of this research is objectivism. We established the probabilistic relationship between the age of girls at menarche as the independent variable and the probability of having menarche at that age as the dependent variable. So, the epistemological position of this research is positivism.

To fit the distributional pattern of age at menarche of Nepalese girls, simulation in Microsoft Excel was used to minimize the sum of the square of error (SSE) using the solver function. To test the validity and suitability of proposed models, the negative log-likelihood (NLL) value of the probability distribution, Akaike's information criteria (AIC), Bayesian information criteria (BIC), chi-square test statistics, SSE, and coefficient of determination (R^2) between the observed and theoretical age of menarche have been applied. The mathematical expressions of the proposed models are discussed as follows.

Logistic Probability Distribution

If X denotes the age at menarche of girls, then the probability density function (PDF) and the cumulative distribution function (CDF) of the logistic distribution are given as:

$$f_1(x) = \frac{e^{-\frac{x-\gamma}{\beta}}}{\beta\left(1+e^{-\frac{x-\gamma}{\beta}}\right)^2} \tag{1}$$

$$F_1(x) = \frac{1}{1+e^{-\frac{x-\gamma}{\beta}}} \tag{2}$$

where $\gamma > 0$ is any arbitrary location parameter, and $\beta > 0$ is the scale parameter of the distribution. According to Johnson et al. (1995), the mean and variance of the distribution are computed by using the relation as "γ" and $\frac{\beta^2\pi^2}{3}$ respectively.

Log-Logistic Probability Distribution

If X denotes the age at menarche of Nepalese females that follow the Log-Logistic (LLog) distribution with probability density function (PDF) and the cumulative distribution function (CDF) with three parameters α, β and γ is given as

$$f_2(x) = \frac{\frac{\alpha}{\beta}\left(\frac{x-\gamma}{\beta}\right)^{\alpha-1}}{\left(1+\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right)^2}, \text{for } x > \gamma \tag{3}$$

$$F_2(x) = \frac{\left(\frac{x-\gamma}{\beta}\right)^{\alpha}}{\left(1+\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right)}, \text{for } x > \gamma \tag{4}$$

where $\alpha > 0$ is the shape parameter, and $\beta > 0$ is the scale parameter. The third parameter γ is a threshold parameter. When the shape parameter α is greater than one, LLog distribution has a uni-model. The basic properties of LLog distribution are studied by (Kleiber and Kotz 2003; Lawless, 2003; Ashkar and Mahdi, 2006). The k^{th} order moments for two-parameter LLog distribution are derived by Tadikamalla (1980) for $k < \alpha$, and the moment had been derived and expressed as

$$E(X^k) = \frac{k\,\pi\,\beta^k}{\alpha \sin\frac{k\pi}{\alpha}} \tag{5}$$

Skew Log-Logistic Distribution

Again, let X be the age of menarche of girls that follow the skew log-logistic distribution proposed by Gaire et al. (2019) and detail studied by Gaire and Gurung (2024). The SLLog distribution's PDF and CDF are as follows:

$$f_3(x) = \frac{2\alpha}{\beta} \frac{\left(\frac{x-\gamma}{\beta}\right)^{2\alpha-1}}{\left(1+\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right)^3} \text{ for } x > \gamma \tag{6}$$

$$F_3(x) = 1 - \frac{2}{1+\left(\frac{x-\gamma}{\beta}\right)^{\alpha}} + \frac{1}{\left(1+\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right)^2} \tag{7}$$

This SLLog distribution was also used to fit the distributional pattern of age at menopause by Gaire et al. (2023), to fit the distributional pattern of age at first marriage data by Gaire et al. (2024a), and to fit age-specific fertility rate by Gaire et al. (2024b).

Model Validation Tools

To test the validity and suitability of the proposed model for the data fitting of age at menarche for Nepalese girls, AIC, BIC, the chi-square test of goodness of fit, SSE, and R^2 have been used. These validation tools were also used to validate their models by Gaire et al. (2022) and Gaire (2023). The chi-square test statistics are calculated as follows:

$$\chi^2 = \sum_{i=0}^{n} \left(\frac{(O_i - E_i)^2}{E_i}\right) \tag{8}$$

where O_i is the observed number of girls having menarche at different ages and E_i is the expected or theoretical number of girls having menarche at different ages obtained from the fitting distribution for Nepalese girls.

Similarly, the formula of AIC and BIC for the fitted models is given as

$$AIC = 2k - 2\,LL \quad (9)$$

$$BIC = k\,Log(n) - 2\,LL \quad (10)$$

where k is the number of parameters associated with the probability distribution. The constant n is the number of observations, and LL is the log-likelihood function at the maximum likelihood estimate for that distribution. The optimization technique was used to fit the model using the solver function available in Microsoft Excel to minimize the sum of squares of error (SSE).

Results and Discussion

The probabilistic distributional pattern of the menarche data of Nepalese girls was fitted by using three probability models for different birth cohorts of girls. The parameters estimated by the proposed models, as well as other statistical results, are presented. The results of the age at menarche data for earlier and intermediate birth cohorts are presented in Table 2. The results of the latest birth cohorts and total birth cohorts are presented in Table 3.

Table 2: For Early (before 1970) and intermediate birth cohorts (1970-1979)

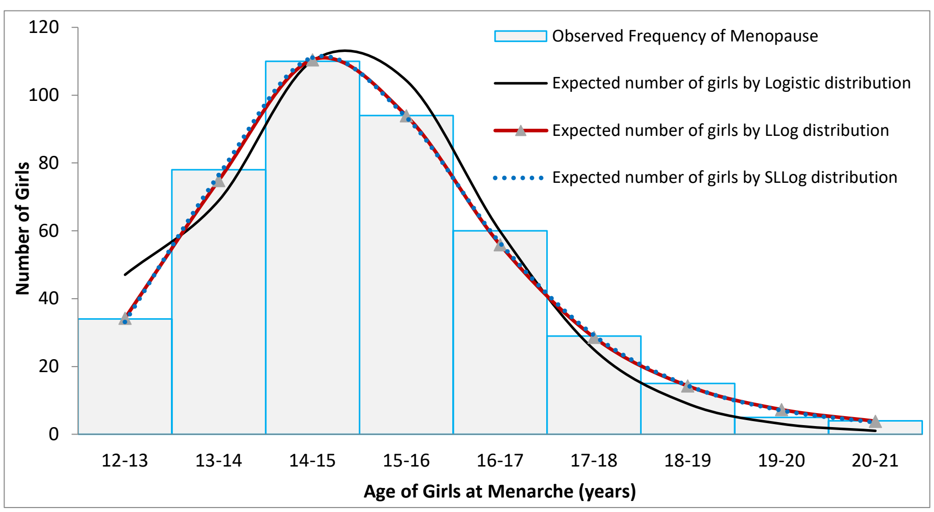
Parameters and Statistics	Logistic		Log-Logistic		Skew Log-Logistic	
	Early	intermediate	Early	intermediate	Early	intermediate
α	-	-	6.246	7.624	13.175	14.301
β	0.908	0.824	6.033	6.564	13.976	13.825
γ	14.901	14.661	8.921	8.136	0.000	0.000
Df	4	4	5	3	5	3
χ^2	14.263	12.202	4.320	1.251	**2.591**	**1.524**
R^2	0.966	0.979	0.995	0.998	**0.997**	**0.999**
NLL	-27.073	-29.486	-24.379	-26.044	**-19.022**	**-25.856**
AIC	58.147	62.972	54.759	58.087	**44.045**	**57.712**
BIC	58.541	63.367	55.350	58.679	**44.283**	**58.304**
SSQ	420.004	327.509	62.202	24.650	**34.338**	**22.396**

Table 3: Parameter and values of different test statistics for Later (1980 and later) and total girls

Parameters and Statistics	Logistic		Log-Logistic		Skew Log-Logistic	
	Later	Total	Later	Total	Later	Total
α	-	-	6.224	6.231	15.317	14.358
β	0.694	0.817	4.548	5.378	12.443	13.753
γ	14.250	14.586	9.742	9.250	1.104	0.000
Df	3	4	2	5	2	5
χ^2	12.973	58.435	2.671	3.084	**1.238**	**1.529**
R^2	0.975	0.969	0.998	0.999	**0.999**	**0.999**
NLL	-26.645	-29.964	-22.539	-25.879	**-22.927**	**-26.137**
AIC	57.289	63.929	51.078	57.758	**51.855**	**56.274**
BIC	57.448	64.323	51.316	58.350	**52.093**	**56.669**
SSQ	418.774	4281.785	34.253	244.776	**15.221**	**105.689**

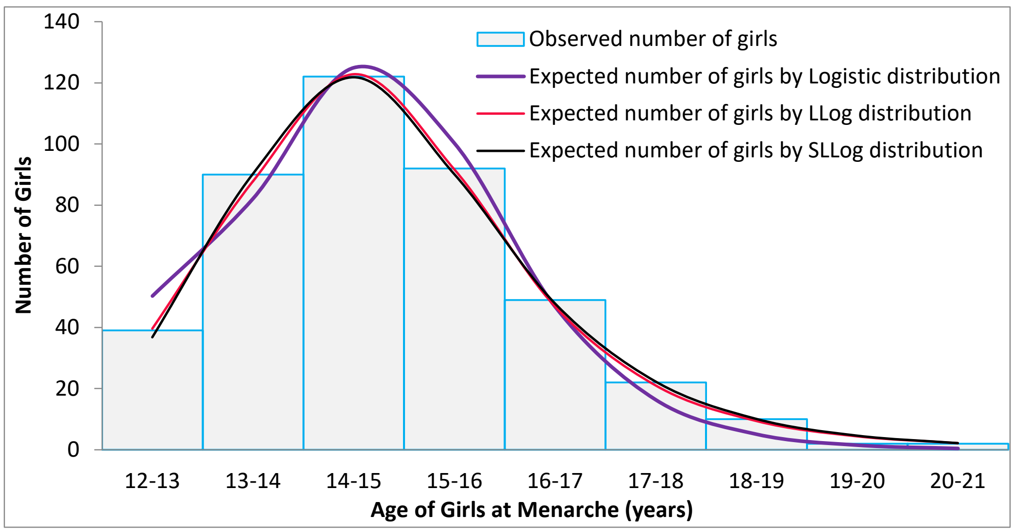
The computed values of negative log-likelihood (NLL), AIC, BIC, chi-square test statistics, R^2, and SSE depict that the proposed SLLog model significantly better fits the age distribution at Nepalese girls' menarche. The calculated value of chi-square, AIC, BIC, and SEE for SLLog distribution for all birth cohorts are lower than those of other distributions, and the NLL and R^2 are found to be maximum. Multiple validation tools confirm that the proposed SLLog model better fits Nepalese girls' distributional age pattern at menarche.

Figure 1 Observed and fitted numbers of girls at the menarche for earlier cohorts (before 1970)



Therefore, the SLLog model is proposed better to fit the female's age at menarche data, and it helps describe the age pattern of girls at menarche. Among them, the SLLog model is statistically significant and best for describing the age at menarche of Nepalese girls. However, in the case of the recent birth cohort, the LLog model seems to fit the data better than the SLLog model. The degree of freedom (df) is computed for the significance test of different test statistics.

Figure 2 Observed and fitted numbers of girls at the age of menarche for the intermediate cohort (1970-1979)



Graphical illustrations show the close association between the observed and expected distribution of age at menarche for earlier (figure 1), intermediate (figure 2), latest (figure 3), and total (figure 4) birth cohorts, which provided a good approximation for describing the distribution of females according to age at menarche.

Figure 3 Observed and fitted numbers of girls at the age of menarche for the latest birth cohort (1980 and later)

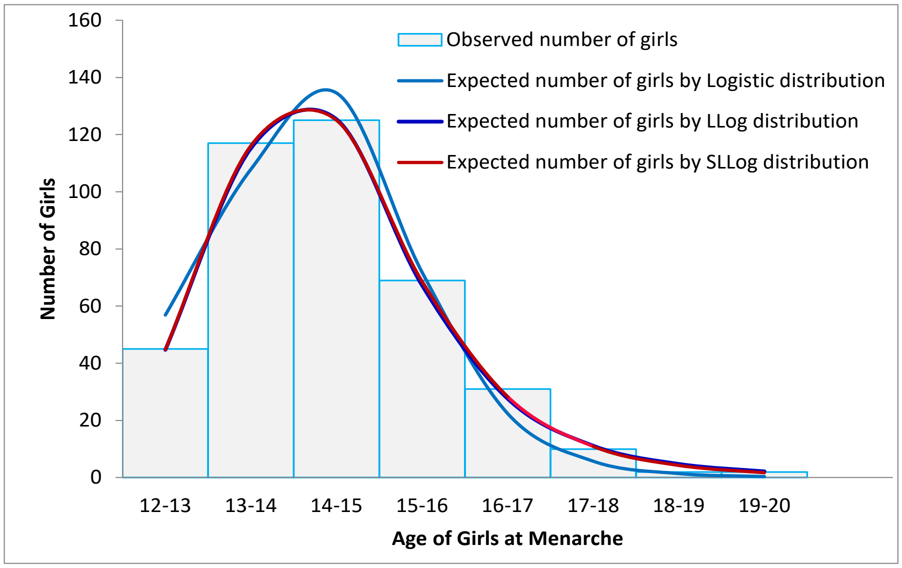
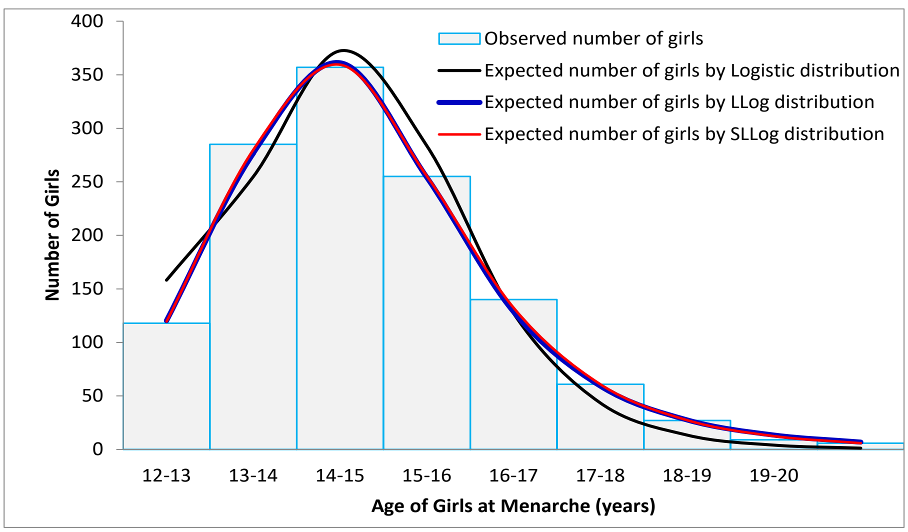


Figure 4 Observed and fitted numbers of girls at the age of menarche for the total birth cohorts.



Construction of Menarchial Life-Table

Here, the observed distributional pattern is a right-skewed curve. So, the SLLog Model has been proposed and used to describe the distributional age pattern at Nepalese girls' menarche. Since this distribution's density and hazard rate function is quickly increasing, it gains maximum and decreases to form a right-tailed curve. The menarchial life table is constructed using the results of a probabilistic model fitting. In this section, the empirical results of probability models used to fit the distributional pattern of age at menarche are used to construct the Menarchial life table. All the procedures are adopted from the concept of mortality life-tale. It is assumed that a menarche is a universal event that every girl undergoes. As such, it parallels mortality life tables, where the number of girls who reached menarche at a certain age are treated in the same way methodologically as age-specific death rates in mortality tables. By fitting the SLLog distribution model, the proportion of girls not having menarche in a specified age or age group is the proportion of "surviving." The following procedure will be used to construct the Menarchial life table.

After fitting the SLLog distribution, suppose F_x is the empirical result of the proportion of girls who experienced menarche at age x year or less and $\mathrm{S_x}$ is the proportion survived or not having menarche at age x where ($\mathrm{S_x} = 1 - \mathrm{F_x}$). From the concept of the mortality life table technique, let l_0 is the cohort of girls of the Menarchial life table. Then $\mathrm{l_x} = \mathrm{l_0 S_x}$. The probability of experiencing menarche between x and x+1 year of age is obtained by using the expression as

$$q_x = \frac{\mathrm{F_{x+1} - F_x}}{\mathrm{1 - F_x}} \tag{11}$$

Table 4: Menarchial Life Table by Using SLLog Distribution for Earlier Cohorts (Before 1970)

Age of Girls	M_x	S_x	l_x	q_x	L_x	T_x	e_x
0 – 12	0.0140	1	100000	0.064	1191603.77	1508431.29	**15.084**
12 – 13	0.0773	0.9860	98600.63	0.193	95434.54	316827.53	3.213
13 – 14	0.2556	0.9227	92268.46	0.348	83354.67	221392.99	2.399
14 – 15	0.5146	0.7444	74440.89	0.449	61492.93	138038.31	1.854
15 – 16	0.7325	0.4854	48544.97	0.492	37645.77	76545.38	1.577
16 – 17	0.8641	0.2675	26746.57	0.502	20167.53	38899.61	1.454
17 – 18	0.9323	0.1359	13588.49	0.496	10179.61	18732.08	1.379
18 – 19	0.9659	0.0677	6770.74	0.485	5090.39	8552.47	1.263
19 – 20	0.9824	0.0341	3410.04	0.471	2583.55	3462.08	1.015
20 – 21	0.9907	0.0176	1757.06	-	878.53	878.53	0.500

Other symbols, such as L_x is the person-year attained menarche by the cohort of girls at age x, T_x is the total person-year attained menarche by the cohort of girls after age x, and e_x is the average number of years expected to attain menarche at age x have been computed by using the following formula as in the mortality life table

$$L_x = \frac{k(l_x + l_{x+k})}{2}, \ T_x = \sum_x^n L_X + L_{X+1} + \cdots + L_{X+n} \tag{12}$$

K is the number of years between years n to n + k

$$T_{x+1} = T_x - L_x \text{ and } e_x = \frac{T_x}{l_x} \tag{13}$$

Table 5: Menarchial Life Table by Using SLLog Distribution for Intermediate Cohorts (1970-79)

Age of Girls	M_x	S_x	l_x	q_x	L_x	T_x	e_x
0 – 12	0.0136	1	100000	0.0733	1191839	1482731	**14.827**
12 – 13	0.0859	0.9864	98639.87	0.2307	95022.74	290892.08	2.949
13 – 14	0.2968	0.9141	91405.61	0.4047	80861.36	195869.34	2.143
14 – 15	0.5814	0.7032	70317.10	0.5028	56088.03	115007.98	1.636
15 – 16	0.7919	0.4186	41858.96	0.5367	31335.88	58919.95	1.408
16 – 17	0.9036	0.2081	20812.79	0.5396	15227.50	27584.08	1.325
17 – 18	0.9556	0.0964	9642.21	0.5300	7040.64	12356.58	1.282
18 – 19	0.9791	0.0444	4439.07	0.5159	3262.70	5315.94	1.198
19 – 20	0.9899	0.0209	2086.33	0.5004	1548.20	2053.25	0.984
20 – 21	0.9950	0.0101	1010.08	-	505.04	505.04	0.500

Table 6: Menarchial Life Table by Using SLLog Distribution for recent cohorts (1980 and after)

Age of Girls	M_x	S_x	l_x	q_x	L_x	T_x	e_x
0 – 12	0.0134	1	100000	0.0996	1191980	1437470	**14.375**
12 – 13	0.1116	0.9866	98663.41	0.3259	93750.21	245489.08	2.488
13 – 14	0.4012	0.8884	88837.02	0.5205	74360.23	151738.87	1.708
14 – 15	0.7129	0.5988	59883.44	0.5959	44297.98	77378.63	1.292
15 – 16	0.8840	0.2871	28712.51	0.6086	20156.95	33080.66	1.152
16 – 17	0.9546	0.1160	11601.39	0.5989	8071.05	12923.70	1.114
17 – 18	0.9818	0.0454	4540.71	0.5822	3181.02	4852.65	1.069
18 – 19	0.9924	0.0182	1821.33	0.5638	1291.15	1671.64	0.918
19 – 20	0.9967	0.0076	760.97	-	380.49	380.49	0.500

Table 7: Menarchial Life Table by Using SLLog Distribution for recent cohorts (1980 and after)

Age of Girls	M_x	S_x	l_x	q_x	L_x	T_x	e_x
0 – 12	0.0153	1	100000	0.0809	1190817	1473664	**14.737**
12 – 13	0.0950	0.9847	98469.54	0.2459	94485.02	282846.75	2.872
13 – 14	0.3176	0.9050	90500.51	0.4185	79371.30	188361.73	2.081
14 – 15	0.6032	0.6824	68242.09	0.5112	53963.23	108990.43	1.597
15 – 16	0.8060	0.3968	39684.38	0.5415	29541.88	55027.20	1.387
16 – 17	0.9110	0.1940	19399.39	0.5426	14147.23	25485.32	1.314
17 – 18	0.9593	0.0890	8895.07	0.5321	6481.88	11338.08	1.275
18 – 19	0.9810	0.0407	4068.69	0.5176	2986.13	4856.20	1.194
19 – 20	0.9908	0.0190	1903.56	0.5019	1410.93	1870.07	0.982
20 –21	0.9954	0.0092	918.29	-	459.14	459.14	0.500

The mean age of menarche estimated using the life table technique for Nepalese girls is 14.737 years. The mean age of menarche for different cohorts has been estimated as 15.08, 14.827, and 14.375 for earlier cohorts (before 1970), intermediate cohorts (1970-1979), and recent cohorts (after 1980), respectively. The mean age at menarche for Nepalese girls is found to be decreasing for recent cohorts of girls than that of earlier cohorts of girls.

Conclusion

Menarche is one of the first and most important indicators of a woman's fertility life. It is a biological event in every girl's life; however, its timing is influenced by several biological and socioeconomic factors. Despite the lack of a universal model to describe the distributional patterns of the age of menarche, researchers are continuously seeking suitable models. This manuscript used two models to describe the distributional pattern of age at menarche in girls. The SLLog model better fits the Nepalese menarche data than the LLog and Logistic models. The result of probability fitting of age at menarche is applied to construct the menarchial life tables for different birth cohorts of Nepalese girls' menarche data. Also, the mean age of menarche obtained from the fitted model is consistent with the observed data. Further, the Menarchial life table of girls also shows that the expected age of girls to have a menstrual period at birth is found to be 15.08, 14.827, and 14.375 years, respectively, for earlier, intermediate, and recent birth cohorts. These findings were consistent with the decreasing mean age of menarche around the globe due to advancement of science and technology and the improvement in girls' nutritional condition. These findings can be helpful to policymakers, planners, and health personnel interested in women's health. Health personnel can help girls who are struggling with the emotional and psychological challenges of early menarche. The difference in the MAM for different sub-populations of Nepal has several implications for policymakers, planners, and health personnel. It helps to be aware of the potential health consequences of early or late menarche. It can play a role in reducing health risks by promoting healthy eating and exercise habits among girls and providing access to comprehensive sexual and reproductive health education for girls. By implementing policies and programs that promote healthy lifestyles and provide girls with access to comprehensive sexual and reproductive health education and support services, policymakers, planners, and health personnel can help to reduce the health risks associated with early menarche.

References

Akther, A., and Islam, R. (2012). Study on age at menarche of school-going girls in Bangladesh, *American Journal of Medicine and Medical Sciences 2*(2), 4-9.

Aryal, T. R. (2005). A probability model to describe the distribution of age at menarche in Nepal. *Nepal Journal of Science and Technology*, 6(1), 47-52.

Aryal, T. R. (2011). Age at menarche and its relation to ages at marriage, first-birth, and menopause among rural Nepalese females. *Nepal Journal of Science and Technology*, *12*, 276-285.

Ashkar, F., and Mahdi S. (2006). Fitting the log-logistic distribution by generalized moments. *Journal of Hydrology*, 328 (3-4), 694-703.

Cameron, N., and Nagdee, I. (1996). Menarchial age in two generations of South African Indians. *Annals of human biology*, *23*(2), 113-119.

De Sanctis, V., Rigon, F., Bernasconi, S., Bianchin, L., Bona, G., Bozzola, M., ... and Perissinotto, E. (2019). Age at menarche and menstrual abnormalities in adolescence: does it matter? The evidence is from a large survey among Italian secondary schoolgirls. *The Indian Journal of Pediatrics*, *86*, 34-41.

Gaire, A. K. (2023). Skew Lomax distribution, parameter estimation, its properties, and applications. *Journal of Science and Engineering*, 10, 1-11.

Gaire, A. K., and Aryal, R. (2015). Inverse Gaussian model to describe the distribution of age-specific fertility rates of Nepal. *Journal of Institute of Science and Technology*, 20(2), 80-83.

Gaire, A. K., and Gurung, Y. B. (2024). Skew Log-Logistic distribution: Properties and Application. *Statistics in Transition New Series*, 25(1), 43-62.

Gaire, A. K., Gurung, Y. B. and Bhusal, T. P. (2023). Stochastic modeling of age at menopause for Nepalese women and development of menopausal life table. *Global Health Economics and Sustainability*. 1(2), 1-11.

Gaire, A. K., Gurung, Y. B., and Bhusal, T. P. (2024a). Age at First Marriage of Nepalese Women: A Statistical Analysis (Status, Differential, Determinants, and Distributional Pattern). *Journal of Population and Social Studies*, *32*, 308–328.

Gaire, A. K., Gurung, Y. B., and Bhusal, T. P. (2024b). Evolution of Fertility Models: A Survey on Mathematical Models of Age-specific Fertility with Application. *Global Health Economics and Sustainability*, 2(4).

Gaire, A. K., Thapa, G. B., and KC, S. (2019). *Preliminary results of Skew Log-logistic distribution, properties, and application. Proceeding of the 2nd International Conference on Earthquake Engineering and Post Disaster Reconstruction Planning* (ICEE-PDRP-2019), 25–27 April 2019, Bhaktapur, Nepal, 37-43.

Gaire, A. K., Thapa, G. B., and KC, S. (2022). Mathematical Modeling of Age-Specific Fertility Rates of Nepali Mothers. *Pakistan Journal of Statistics and Operation Research*, 18(2) 417-426.

Garenne, M. (2020a). Estimating the median age at menarche with a Logit model: Application to African DHS surveys. *African Population Studies*, *34*(1) 5060-5070).

Garenne, M. (2020b). Trends in age at menarche and adult height in selected African countries (1950–1980). *Annals of Human Biology*, *47*(1), 25-31.

Ghimire, M., and Ghimire, M. (2014). Menarche and Its determinants in adolescent girls. *Journal of Lumbini Medical College*, *2*(1), 10-13.

Johnson, N. L., Kotz, S., and Balakrishnan, N. (1995). *Continuous univariate distributions, volume 2* (Vol. 289). John Wiley and Sons.

Khan, H. G., and Hallad, S. J. (1994). Age at menopause and menopausal transition: Perspectives of Indian rural women. *Journal of Midlife Health*, 1-21.

Kisch, E. H. (1910). *The sexual life of a woman in its physiological, pathological, and hygienic aspects*. Rebman Publication, London, UK.

Kleiber, C., and Kotz, S. (2003). *Statistical size distribution in economics and actuarial sciences*. John Wiley and Sons, 470, 694-703.

Lacroix, A. E., Gondal, H., Shumway, K. R., and Langaker, M. D. (2022). Physiology, menarche. In *StatPearls [Internet]*. StatPearls Publishing.

Lawless, J. F. (2003). *Statistical models and methods for lifetime data. Vol. 362.* John Wiley and Sons. New York, USA.

Lehmann, A., Scheffler, C., and Hermanussen, M. (2010). The variation in age at menarche: an indicator of historic developmental tempo. *Anthropologischer Anzeiger*, *68*(1), 85–99.

Mascie-Taylor, C. G. N., and Boldsen, J. L. (1986). Recalled the age of menarche in Britain. *Annals of human biology*, *13*(3), 253-257.

Raji, Y., Osonuga, O. A., Shittu, O. I., Akinsomisoye, O. S., Togun, V. A., and Azeez, M. O. (2006). Age at menarche and its predicting factors in cities of Ibadan and Ogbomoso of Southwestern Nigeria. *Journal of Medical Science*, *6*(5), 772-778.

Tadikamalla, P. R. (1980). A look at the Burr and related distributions. *International Statistical Review*, 337-344.